QUESTION BANK

Unit 1 (CO1)

To Find C. F. (Complementary Function)

1. $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$
2. $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$
3. $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 8y = 0$, Under the conditions y(0) = 0, y'(0) = 0 and y''(0) = 2.
4. $(D^2+1)^2(D-1)y = 0$
5. $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0 \; given\ that\ when\ t = 0, x = 0\ and\ x' = 0$

To Find P.I. (Particular Integral)

When $R = e^{ax}$

6. Solve $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 8$
7. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{-2x} + e^{-3x}$
8. Solve $(D-1)^2(D+2) = e^{-2x} + 2\sinh x$.

When $R = Sinax\ or\ Cosax$

9. Solve $(D^2+4)y = \sin 3x + \cos 2x$
10. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$
11. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$ and find the value of y when $x = \frac{\pi}{2}$ being given that $y = 3, \frac{dy}{dx} = 0\ when\ x = 0.$
12. Solve$(D^2+5D-6)y = \sin 4x \sin x$.

When $R = x^n$

13. Solve$(D^2+5D+4)y = x^2 + 7x + 9$.
14. Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$.

When $R = e^{ax}\varphi(x)$

15. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$.
16. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \cos x$.
17. Solve $(D^2-4D+4)y = 8\,x^2 e^{2x} \sin 2x$.
18. A body execute damped forced vibration given by the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin \omega t.$$

Solve the equation for both the cases, when $\omega^2 \neq b^2 - k^2\ and\ \omega^2 = b^2 - k^2$.

When $R = \varphi(x)$

19. Find the complete solution of $(D^2 + a^2)y = \sec ax$.

20. Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$.

Homogeneous Linear Differential Equations

Euler- Cauchy Equations-

21. Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$.

22. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = \sin(logx)$.

23. Solve$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = \text{x}(logx)$.

24. Solve$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3e^x$..

Legendre's Linear Differential Equation

25. Solve$(3x + 2)^2\frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

26. Solve$(x + 1)^2\frac{d^2y}{dx^2} + (x + 1)\frac{dy}{dx} + y = 4\cos\{\log(x + 1)\}$.

27. Solve$(x + 1)^2\frac{d^2y}{dx^2} + (x + 1)\frac{dy}{dx} = (2x + 3)(2x + 4)$.

28. Solve$(2x + 3)^2\frac{d^2y}{dx^2} - 2(2x + 3)\frac{dy}{dx} - 12y = 6x$.

Simultaneous Linear Differential Equations

29. Solve the Simultaneous Linear Differential Equations$\frac{dy}{dt} + 2x + y = 0, \frac{dx}{dt} + 5x - 2y = t$.

given that x = y = 0 when t = 0.

30. Solve the Simultaneous Linear Differential Equations$\frac{dx}{dt} = -uy, \frac{dy}{dt} = ux$. Also show that the point (x, y) lies on a circle.

31. Solve $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}, \frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = \sin 2t$.

32. Solve $\frac{dx}{dt} + y = sint, \frac{dy}{dt} + x = \cos t; given\ that\ x = 2\ and\ y = 0\ when\ t = 0$

Linear Differential Equations of Second Order

When Part of Complementary Function is known

33. Solve $\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - (1 - \cot x)y = e^x sinx$.

34. Solve $(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} - ycos\ x = 0\ of\ which\ y = x\ is\ a\ solution$.

Normal Form (Removal of First Derivative)

35. Solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$

36. Solve $\frac{d}{dt}\left[cos^2x\ \frac{dy}{dx}\right] + cos^2x.y = 0$

Method of Changing the independent variable.

37. Solve $\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y\, cosec^2 x = 0$ by changing the independent variable.

38. Solve $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (\sin^2 x)y = \cos x - cos^3 x$ by changing the independent variable.

39. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ by changing the independent variable.

Equations that do not contain 'x' directly

40. Solve $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$

Equations that do not contain 'y' directly

41. Solve $x\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} = 0$

42. Solve $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}$

43. Solve $\frac{d^2y}{dx^2} = \left[1 - \left(\frac{dy}{dx}\right)^2\right]^{1/2}$

Differential equations of the type Solve $\frac{d^2y}{dx^2} = f(y)$

44. Solve $\frac{d^2y}{dx^2} = \sqrt{y}$, *under the condition* $y = 1, \frac{dy}{dx} = \frac{2}{\sqrt{3}}$ *at* $x = 0$.

45. Solve $\frac{d^2y}{dx^2} = sec^2 y \tan y$ *under the condition* $y = 0$ *and* $\frac{dy}{dx} = 1$ *when* $x = 0$.

46. Solve $\frac{d^2y}{dx^2} = 2(y^3 + y)$, *under the condition* $y = 0, \frac{dy}{dx} = 1$, *when* $x = 0$.

Differential equations of the type Solve $\frac{d^2y}{dx^2} = f(x)$

47. Solve $\frac{d^2y}{dx^2} = x^2 \sin x$

Method of Variation of Parameter (Most Important Topics)

48. Solve
    i. $\frac{d^2y}{dx^2} + y = cosec\, x$
    ii. $\frac{d^2y}{dx^2} + y = tan\, x$
    iii. $\frac{d^2y}{dx^2} + y = sec\, x$

49. Apply method of variation Parameter to find the general solution of $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 3x = \frac{e^t}{1+e^t}$

50. Using the following simultaneous equation, solve$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$.

Unit 2 (CO 2)

1. Find the Laplace Transform of the following

i. $L(1+\cos 2t)$.

ii. $f(t)=\begin{cases}0, & 0<t<1\\ t, & 1<t<2\\ 0, & t>2\end{cases}$

iii. $L(e^{3t}\cos 5t)$

iv. $if\ L(cos^2 t)=\frac{s^2+2}{s(s^2+4)}, Find\ L(cos^2 at)$

v. $L(t\ e^{-t} sin2t)$

vi. $L\left\{\frac{\cos at-\cos bt}{t}\right\}$

vii. $L\left\{\frac{e^{at}-\cos bt}{t}\right\}$

viii. $L\left\{\frac{1-\cos t}{t^2}\right\}$

ix. $L\left\{\int_0^\infty \frac{(e^{-at}-e^{-bt})}{t}\ dt\right\}$

x. Show that $L\left\{\int_0^\infty \frac{\sin t}{t} dt=\frac{\pi}{2}\right\}$

2. Compute L {F (t)}, if $F(t)=\begin{cases}\sin\omega t, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$

3. Draw the graph of the following periodic function and find its Laplace Transform:

$$F(t)=\begin{cases}t & for\ 0<t<a\\ 2a-t & for\ \ a<t<2a\end{cases}$$

4. Express the following function in terms of unit step functions and find its Laplace transform:

$$F(t)=\begin{cases}t-1, & 1<t<2\\ 3-t, & 2<t<3\end{cases}$$

5. Find $L^{-1}\left\{\frac{6}{2s-3}-\frac{3+4s}{9s^2-16}+\frac{8-6s}{16\,s^2+9}\right\}$

6. Find $L^{-1}\left\{\frac{1}{s^2-6s+10}\right\}$

7. Find $L^{-1}\left\{\frac{e^{-s}-3\,e^{-3s}}{s^2}\right\}$

8. Evaluate $L^{-1}\left\{\frac{2\,s^2-1}{(s^2+4)(s^2+1)}\right\}$

9. Obtain the inverse Laplace transform of $cot^{-1}\left(\frac{s+3}{2}\right)$

10. Evaluate $L^{-1}\left\{log\,\frac{s+a}{s+b}\right\}$

11. Evaluate $L^{-1}\left\{tan^{-1}\frac{2}{s^2}\right\}$

12. Evaluate $L^{-1}\left\{cot^{-1}\frac{s}{2}\right\}$

13. Use the convolution theorem to find

i. $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+1)}\right\}$

ii. $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

iii. $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$

14. Using Heaviside's expansion formula find

i. $L^{-1}\left\{\frac{2\,s^2-6s+5}{(s^3-6\,s^2+11s-6)}\right\}$

ii. $L^{-1}\left\{\frac{2\,s^2+5s-4}{(s^3+s^2-2s)}\right\}$

15. Solve $(D^2+9)x = 6\cos 3t,\ \ t > 0\ with\ 2\ and\ 0\ for\ values\ of\ x\ and\ Dx\ when\ t = 0.$

16. Solve $(D^2+9)y = \cos 2t\ if\ y(0) = 1,\ \ y\left(\frac{\pi}{2}\right) = -1.$

17. Solve $(D^3-3D^2+3D-1)y = t^2e^t, y(0) = 0, y'(0) = 0, y''(0) = -2.$

18. Solve $(D^3-2D^2+5D)y = 0, y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{8}\right) = 1.$

19. Solve $Dx + y = \sin t; x + Dy = \cos t, if\ x = 2,\ \ y = 0, when\ t = 0.$

20. Solve $\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0;\ \frac{dx}{dt} + 2y = e^{-t}, if\ x(0) = y(0) = 0.$

Unit 3 (CO3)

Fourier series

1. Find the Fourier series to represent $x - x^2\ from\ x = -\pi\ to\ x = \pi.$

$$Deduce\ that\ \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

2. Obtained the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)\ in\ the\ interval\ (0,2\pi) and\ hence\ deduce$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

3. Find the Fourier series expansion for f(x), if

$F(x) = \begin{cases} -\pi & for -\pi < x < 0 \\ x & for\ \ 0 < x < \pi \end{cases}$ $hence\ deduce\ \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

4. Obtained a Fourier series to represent the function $f(x) = |x|, for -\pi < x < \pi\ and\ hence\ deduce$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

5. Obtained Fourier series for the function $F(x) = \begin{cases} \pi x & for\ 0 \le x \le 1 \\ \pi(2 - x) & for\ \ 1 \le x \le 2 \end{cases}$

6. Obtained the Fourier series for the expansion of $f(x) = x\ \sin x$ in the interval $-\pi < x < \pi\ and\ hence\ deduce\ that\ \frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots$

7. Obtained Fourier series of the function $F(x) = \begin{cases} x & for -\pi < x < 0 \\ -x & for\ \ 0 < x < \pi \end{cases}$

8. Obtained Fourier series of the function$F(x) = x + \frac{x^2}{4}, -\pi < x < \pi$

9. Find the Fourier half-range cosine series of the function

$$F(t) = \begin{cases} 2t & for\ 0 < t < 1 \\ 2(2 - t\,) & for\ \ 1 < t < 2 \end{cases}$$

10. Find the Fourier half-range sine series of the function

$$F(t) = \begin{cases} x & for\ 0 < t < 2 \\ (4 - x\,) & for\ \ 2 < t < 4 \end{cases}$$

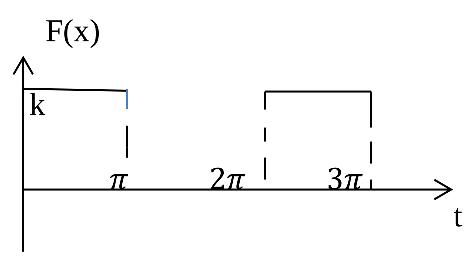
11. Obtained the half- range sine series for the function $F(x) = x^2\ in\ the\ interval\ 0 < x < 3.$

12. Find the Fourier series expansion for f(x), if

$$F(x) = \begin{cases} -k & for -\pi < x < 0 \\ k & for\ \ 0 < x < \pi \end{cases}$$

$$hence\ deduce\ \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

13. Obtained the Fourier series for the square waveform up to 4 terms as shown in the following figure:



Sequence Series

14. Test the convergence of the following series:

- **i.** $1 + 4 + 7 + 10 + \cdots \infty$
- **ii.** $\sum \sqrt{n+1} - \sqrt{n}$
- **iii.** $\sum \frac{1}{n(n+1)(n+2)(n+3)}$
- **iv.** $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$
- **v.** $\sum_{n=1}^{\infty} \frac{1}{n} sin \frac{1}{n}$

D'Alembert's Ratio test

15.

- **i.** Prove that $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots \infty$ convergence.
- **ii.** $\sqrt{\frac{1}{2}}x + \sqrt{\frac{2}{5}}x^2 + \sqrt{\frac{3}{10}}x^3 + \cdots \infty, x > 0.$

16. Find the value of x for which the given series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$

Raabe's test (Higher ratio test)

17. Is the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ converse or diverges?

18. Test the convergence for the series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \cdots$

19. Test the convergence for the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$

GAUSS'S Test

20. Test the convergence for the series $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \cdots$

Cauchy's Integral Test

21. Discuss the convergence of integral $\int_{-2}^{2} \frac{dx}{x^2}$

22. Test the convergence of $\sum_{n=1}^{\infty} \frac{2\, tan^{-1} n}{1+n^2}$

<u>Unit 4 (CO4)</u>

Function of complex Variable, Analytic Functions

1. Determine whether $\frac{1}{z}$ is analytic or not.

2. Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative.

3. Test the analyticity of the function w = sin z and hence derive that: $\frac{d}{dz}(\sin z) = \cos z$.

4. Find the value of C_1 and C_2 such that the function

$$f(z) = x^2 + C_1y^2 - 2xy + i(C_2x^2 - y^2 + 2xy) \text{ is analytic. Also find } f'(z).$$

5. *Discuss the analyticity of the function* $f(z) = z\bar{z}$.

6. Show that the function f (z) defined by $f(z) = \begin{cases} \frac{x^3y^5(x+iy)}{x^6+y^{10}}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$ is not analytic at the origin even though it satisfies Cauchy Riemann equations at the origin.

7. Examine the nature of the function $f(z) = \begin{cases} \frac{x^3y(y-ix)}{x^6+y^2}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$ Prove that

$$\frac{f(z) - f(0)}{z} \to 0 \text{ as } z \to 0 \text{ along any radius vector but not as}$$

$$z \to 0 \text{ in any manner and also that } f(z) \text{is not analytic at } z = 0.$$

Harmonic Function

8. Define a harmonic function and conjugate harmonic function. Find the harmonic conjugate function of the function U(x, y) = 2x (1 – y).

9. Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that f(z) = u + iv is analytic. Also, express f (z) in terms of z.

10. If $\omega = \emptyset + i\varphi$ represent the complex potential for an electric field and $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function $\emptyset$.

11. Find the imaginary part of the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2$.

12. Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express u+ iv as an analytic function of z.

13. Show that $e^x(x \cos y - y \sin y)$is a harmonic function. Find the analytic function for which $e^x(x \cos y - y \sin y)$ is imaginary part.

14. $If\ u - v = (x - y)(x^2 + 4xy + y^2)$ and f(z) = u + iv is an analytic function of z = x + iy, find f(z) in terms of z by Milne Thomson method.

15. $If\ f(z) = u + iv$ is an analytic function of z = x + iy and

$$u - v = e^{-x}[(x - y)\sin y - (x + y)\cos y] find\ f(z).$$

16. $Find\ the\ analytic\ function\ f(z) = u(r,\theta) + i\ v(r,\theta)$ such that

$v(r,\theta) = r^2 cos2\theta - r\ cos\theta + 2.$

17. If f (z) is a harmonic function of z, such that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$

18. Consider the transformation $w = ze^{i\frac{\pi}{4}}$ and determine the region R' in w – plane corresponding to the triangular region R bounded by the lines x = 0, y = 0 and x + y =1 in z-plane.

19. Find the image of the triangle with vertices at i, 1 + i, 1 – i in the z – plane, under the transformation

w= 3z + 4- 2i.

20. Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$.

Bilinear Transformation

21. Find the bilinear transformation which maps the points z = 1, i, -1 into the points w = i, 0, -i. Hence find the image of $|z| < 1$.

22. Find the bilinear transformation which maps the points z = 1, - i, -1 to the points w = i, 0, - i respectively. Show also that transformation maps the region outside the circle $|z| = 1$ into the half-plane $R \geq 0$.

23. Find the bilinear transformation which maps the points z = 0, -1, i onto w = i, 0,∞. Also find the image of the unit circle $|z| = 1$.

24. Find the fixed points and the normal form of the following bilinear transformations.

(a) $w = \frac{3z-4}{z-1}$

(b) $w = \frac{z-1}{z+1}$

Discuss the nature of these transformations.

25. Find two bilinear transformations whose fixed points are 1 and 2.

Unit – 5 (CO5)

1. Find the value of the integral $\int (x + y)dx + x^2ydy$

(a) Along $y = x^2$, having (0, 0), (3, 9) end points.

(b) Along y = 3x between the same points.

Do the values depend upon path?

2. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + 3y^2)dx + 2(x^2 + 3xy + 4y^2)dy$

(a) Along $y^2 = x$

(b) Along $y = x^2$

(c) Along y = x.

3. Evaluate $\int_c (12z^2 - 4iz)dz$ along the curve C joining the points (1, 1) and (2, 3).

4. Evaluate the line integral $\int_c z^2dz$ where C is the boundary of a triangle with vertices 0, -1+ i, 1+ i clockwise.

5. Evaluate $\int_c (z + 1)^2dz$ where C is the boundary of the rectangle with vertices at the points a + ib, -a + ib, a – ib.

6. Verify Cauchy's Theorem for the function $f(z) = e^{iz}dz$ along the boundary of the triangle with vertices at the points 1 + i, -1 + i and -1 – i.

7. Verify Cauchy's Theorem for the function $f(z) = 3z^2 + iz - 4$ along the perimeter of square with vertices $1 \pm i,\ -1 \pm i$.

8. Define Cauchy's Integral Formula.

9. Use Cauchy's integral formula to evaluate $\int_c \frac{z}{(z^2-3z+2)} dz$ where c is the circle $|z - 2| = \frac{1}{2}$.

10. Evaluate the complex integral $\int_c \tan z\, dz\ where\ c\ is\ |z| = 2$.

11. Evaluate $\int_c \frac{e^z}{(z-1)(z-4)} dz\ where\ C\ is\ the\ circle\ |z| = 2$ by using Cauchy's Integral Formula.

12. Evaluate $\int_c \frac{e^z}{(z^2+1)} dz$ over the circle path $|z| = 2$.

13. Evaluate the following integral using Cauchy integral formula

$\int_c \frac{z^2-2z}{(z+1)^2\,(z^2+4)} dz\ where\ c\ is\ the\ circle\ |z| = 10.$

14. Use Cauchy's integral formula to evaluate $\int_c \frac{e^z}{(z+1)^2} dz$ where c is the circle $|z - 1| = 3$.

15. Find the value of $\int_c \frac{\exp(i\pi z)}{(2z^2-5z+2)} dz$, where C is the unit circle with Centre at the origin.

Taylor's Series

16. Expand $\frac{1}{(z^2-3z+2)}$ in the region **(a)** $|z| < 1$ **(b)** $|z| > 2$ **(c)** $1 < |z| < 2$

17. For the function $f(z) = \frac{4z-1}{z^4-1}$, find all Taylor series about the Centre zero.

18. Expand the function $\sin^{-1} z$ in power of z.

19. Expand the function $\tan^{-1} z$ in power of z.

Laurent's Theorem (series)

20. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in the Laurent series valid for

- **i.** $|z-1| > 1$
- **ii.** $0 < |z-2| < 1$

21. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for

- **i.** $1 < |z| < 3$
- **ii.** $|z| > 3$
- **iii.** $0 < |z+1| < 2$
- **iv.** $|z| < 1$

22. Define the singularity of a function. Find the singularity (ties) of the functions

- **i.** $f(z) = sin\frac{1}{z}$
- **ii.** $g(z) = \frac{e^{\frac{1}{z}}}{z^2}$

23. Define the Laurent series expansion of function. Expand $g(z) = e^{\frac{z}{(z-2)}}$ in a Laurent series about the points z = 2.

24. Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ $at\ z = 1, 2, 3\ at\ infinity\ and$ show that their sum is zero.

25. Determine the poles and residue at each pole of the function f(z) = cot z.

26. Evaluate the following integral using residue theorem $\int_c \frac{4-3z}{(z)(z-1)(z-2)} dz\ \ where\ c\ \ is\ the\ circle\ |z| = \frac{3}{2}$.

27. Evaluate $\int_C \frac{12z-7}{(z-1)^2(2z+3)} dz\ , where\ C\ is\ the\ circle$

- **i.** $|z| = 2$
- **ii.** $|z+i| = \sqrt{3}$

28. Using Residue theorem, evaluate $\frac{1}{2\pi i}\int_C \frac{e^{zt}}{(z)^2\ (z^2+2z+2)} dz\ , where\ C\ is\ the\ circle\ |z| = 3.$

29. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)}$

30. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{(a+b\sin\theta)}$ $if\ a > |b|$

31. Evaluate the integral $\int_0^{2\pi} \frac{cos\theta}{(5+4\cos\theta)} d\theta$ by using contour integration.

32. Evaluate the integral $\int_0^{2\pi} \frac{cos3\theta}{(5-4\cos\theta)} d\theta$ by using contour integration.

33. Evaluate the integral $\int_0^{2\pi} \frac{cos3\theta}{(5+4\cos\theta)} d\theta$ by using contour integration.

34. Use the residue theorem to show that $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}}$ $where\ a > 0, b > 0, a > b.$

35. Evaluate $\int_0^{\infty} \frac{\cos mx}{(x^2+1)} dx$

36. Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$

37. Evaluate $\int_{-\infty}^{\infty} \frac{\text{x}\sin\pi x}{(x^2+2x+5)} dx$

38. Using the complex variable techniques, evaluate the integral $\int_{-\infty}^{\infty} \frac{1}{(x^4+1)} dx$