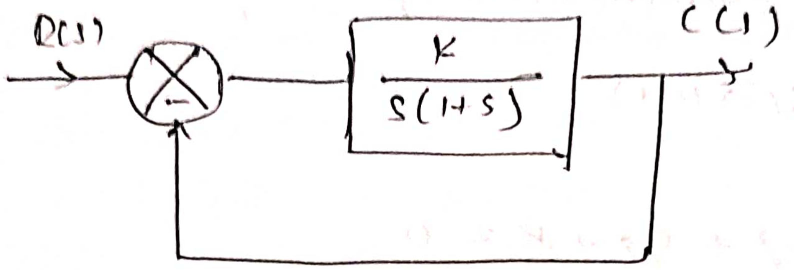
Ques.
(1) For $r(t) = 0.1t$

It is desired that

$e_{ss} \leq 0.005$

For what value of "K" this will take place?



a) $K \geq 20$ b) $K \geq 10$ c) $K \geq 5$ d) $K \geq 50$

Solving

Sol! →

$$R(s) = \frac{0.1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)H(s)}$$

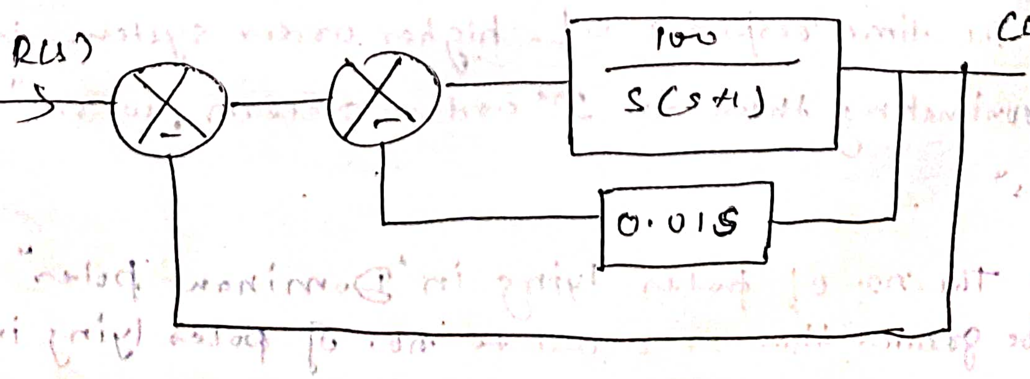
$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \times \frac{0.1}{s^{\cancel{2}}}}{1 + \frac{K}{s(1+s)}}$$

$$e_{ss} = \frac{0.1}{K}$$

$$\frac{0.1}{K} \leq 0.005$$

$$\boxed{K \geq 20}$$

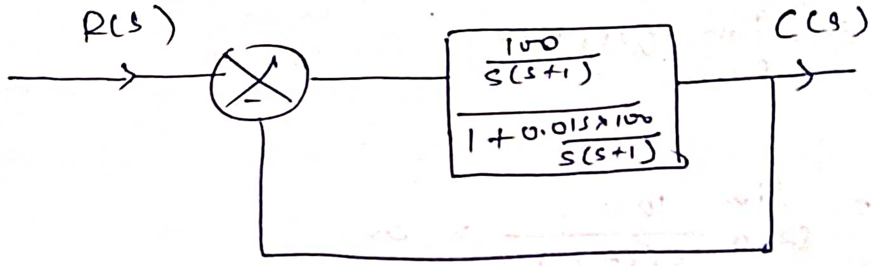
Ques.

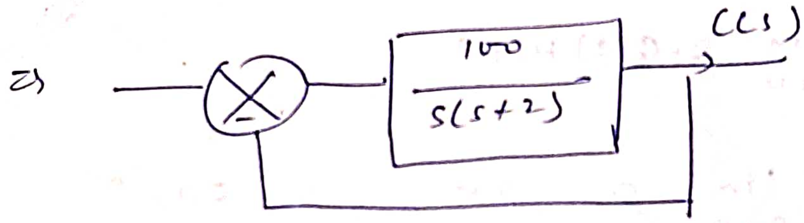


Find e_{ss} for $r(t) = 5 + 2t$.

Sol:- $R(s) = \cancel{\frac{5}{s} + \frac{2}{s^2}}$

$$e_{ss} = \cancel{\lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s)H(s)}}$$





Now $r(t) = 5 + 2t$

$\Rightarrow R(s) = \frac{5}{s} + \frac{2}{s^2}$

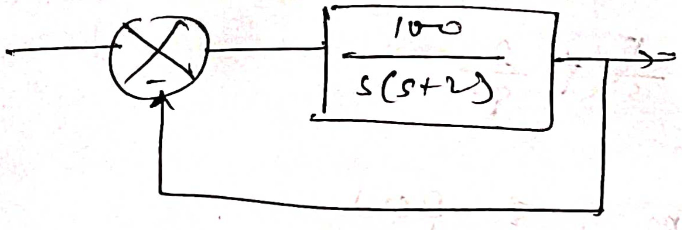
$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot \left(\frac{5}{s} + \frac{2}{s^2}\right)}{1 + \frac{100}{s(s+2)}}$$

$$= \lim_{s \to 0} \frac{5 + \frac{2}{s}}{1 + \frac{100}{s(s+2)}}$$

$$= \lim_{s \to 0} \frac{5\cancel{s} + 2}{s + \frac{\cancel{s} \cdot 100}{\cancel{s}(s+2)}}$$

$$= \frac{2}{\frac{100}{2}} = \frac{4}{100} = \frac{1}{25} \text{ units.}$$

Question



i) Static error Constant :-

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{100}{s(s+2)} = \infty$$

$$K_V = \lim_{s\to 0} s\cdot G(s)H(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot \frac{100}{\cancel{s}(s+2)} = 50$$

$$K_A = \lim_{s\to 0} s^2\cdot G(s)H(s)$$

$$= \lim_{s\to 0} s^{\cancel{2}}\times \frac{100}{\cancel{s}(s+2)} = 0$$

ii) Dynamic Error Constant :-

$$K_0 = \lim_{s\to 0} F(s)$$

As we know

$$F(s) = \frac{1}{1+G(s)H(s)}$$

2) $$K_0 = \lim_{s\to 0} \frac{1}{1+\frac{100}{s(s+2)}}$$

$$= \lim_{s\to 0} \left[\frac{s(s+2)}{s(s+2)+100}\right] = 0$$

$$\Rightarrow \boxed{K_0 = \frac{1}{K_p}}$$

Now :-

$$K_1 = \lim_{s\to 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{d(s)} \frac{s^2+2s}{s^2+2s+100}$$

$$= \frac{(s^2+2s+100)(2s+2) - (s^2+2s)(2s+2)}{(s^2+2s+100)^2}$$

$$\therefore K_1 = \lim_{s\to 0} \frac{d}{ds} F(s) = \frac{(0+0+100)(2) - 0}{(100)^2}$$

$$= \frac{1}{50}$$

$$\Rightarrow \boxed{K_1 = \frac{1}{K_V}}$$

similarly :- $\boxed{K_2 = \frac{1}{K_A}}$

Now the value of "e_{ss}" by using error series :-

$$e_{ss} = \lim_{t\to\infty} \left[K_0 r_0(t) + K_1 r^{\circ}(t) + \frac{K_2}{2!} r^{\circ\circ}(t) + \dots \right]$$

$$r(t) = 5+2t$$

$$\Rightarrow K_0 = 0$$

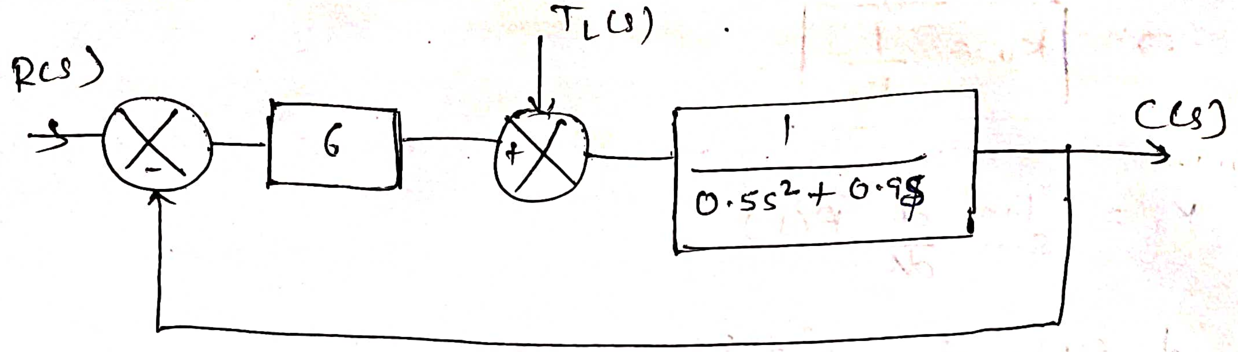
$$r^{\circ}(t) = 0+2 = 2 \rightarrow K_1 = \frac{1}{50}$$

$$r^{\circ\circ}(t) = 0$$

$$\therefore e_{ss} = \lim_{t\to\infty} \left\{ [0 \times (5+2t)] + \left[\frac{1}{50} \times 2\right] \right\}$$

$$\boxed{e_{ss} = \frac{1}{25} \text{ units}}$$

Quest

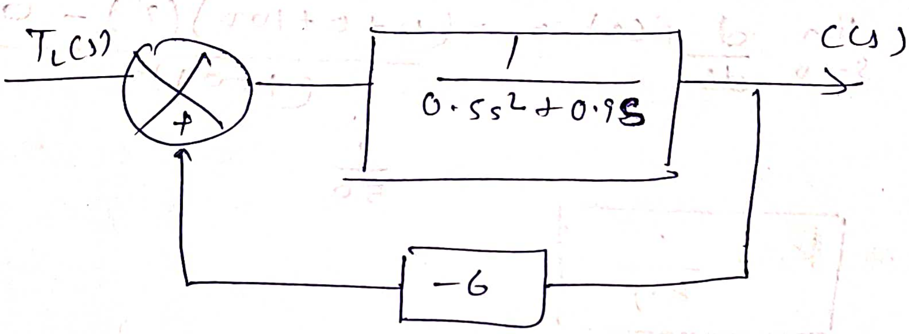


Find the "e_{ss}" when the i/p is held fixed.

and $T_L(s) = 2t$ (N-m) torque is applied

to the system.

Sol^n.



$$\therefore\ e_{ss} = \lim_{s\to 0} s \cdot \frac{T_L(s)}{1 - G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{\cancel{s}\cdot \frac{2}{s^{\cancel{2}}}}{1 + \frac{6}{0.5s^2+0.9s}}$$

$$= \lim_{s\to 0} \frac{2}{s + \frac{6s}{0.5s^2+0.9s}}$$

$$= \lim_{s\to 0} \frac{2}{s + \frac{6}{0.5s+0.9}}$$

$$= \frac{2}{0 + \frac{6}{0.9}} = \frac{2\times \cancel{0.9}^{\,0.3}}{\cancel{6}_{\,3}}$$

$\therefore \boxed{e_{ss} = 0.3 \text{ unit}}$

W.B (Ex-3)

20:- $\frac{G(s)}{1+G(s)} = \frac{a_{n-1}s + a_n}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n}$

$G(s)\left[s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n\right] = (a_{n-1}s + a_n)\left[1 + G(s)\right]$

$G(s)\left[s^n + a_{n-1} s^{n-1} + \dots + \cancel{a_{n-1}s} + \cancel{a_n} - \cancel{a_{n-1}s} - \cancel{a_n}\right] = a_{n-1}s + a_n$

$G(s) = \frac{a_{n-1}s + a_n}{s^n + a_{n-1}s^{n-1} + \dots}$

$\therefore e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{s^{\cancel{2}}}}{1 + \frac{a_{n-1}s + a_n}{s^n + a_{n-1}s^{n-1} + \dots}}$

$= \lim_{s \to 0} \frac{1}{s + \frac{\cancel{s}(a_{n-1}s + a_n)}{\cancel{s}(s^{n-1} + a_{n-1}s^{n-2} + \dots)}}$

$= \frac{1}{0 + \infty}$

$= \frac{1}{\infty} = 0$

31:- $r(t) = 1 + 2t$

$G(s)H(s) = \frac{10(s+1)}{s^2(s+2)}$

$R(s) = \frac{1}{s} + \frac{2}{s^2} = \frac{s+2}{s^2}$

$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{(s+2)}{s^{\cancel{2}}}}{1 + \frac{10(s+1)}{s^2(s+2)}}$

$$e_{ss} = \lim_{s\to 0} \frac{2(s+2)}{s + \cancel{s}\frac{10(s+1)}{s^{\cancel{2}}(s+2)}}$$

By taking limit

$$e_{ss} = \frac{0+2}{0 + \frac{10\times(0+1)}{0(0+2)}}$$

$$= \frac{2}{\infty} = 0$$

34!.

$$r(t) = 2 + 3t + 4t^2$$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{4\times 2}{s^3}$$

$$R(s) = \frac{2s^2 + 3s + 8}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\times \frac{2s^2+3s+8}{s^{\cancel{3}2}}}{1 + \frac{10}{s^2(4+s)}}$$

$$= \lim_{s\to 0} \frac{2s^2+3s+8}{s^2 + \frac{10}{4+s}}$$

Taking limit

$$\frac{8}{0 + \frac{\cancel{10}5}{\cancel{4}2}}$$

$$= \frac{8\times 2}{5} = \frac{16}{5} = 3.2 \quad \underline{\text{Ans}}$$

38.

$$r(t) = (1-t^2)\,3u(t)$$

$$r(t) = (1-t^2)\,3$$

$$= 3 - 3t^2$$

$$R(s) = \frac{3}{s} - \frac{3\times 2}{s^3}$$

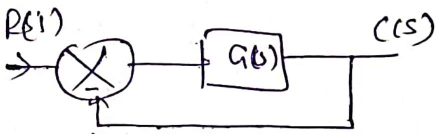
$$e_{ss} = \lim_{s\to 0} \frac{s\times\left(\frac{3}{s} - \frac{6}{s^3}\right)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0}\left[\frac{\cancel{s}\times\frac{3}{\cancel{s}}}{1+G(s)H(s)} - \frac{\cancel{s}\times\frac{6}{\cancel{s^3}\,s^2}}{1+G(s)H(s)}\right]$$

$$= \frac{3}{1+\lim_{s\to 0} G(s)H(s)} - \frac{6}{s^2 + \lim_{s\to 0} s^2 G(s)H(s)}$$

$$= \frac{3}{1+K_p} - \frac{6}{K_A}$$

40. $\cancel{H(s) = \frac{2(s+3)}{(s+2)(s+4)}}$

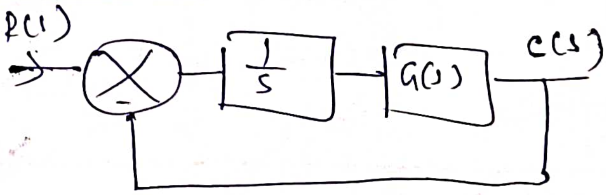


43.

$$e_{ss} = 0.20$$

$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$



$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s}}{1+G(s)}$$

$$0.2 = \frac{1}{1+\lim_{s\to 0} G(s)}$$

$$\frac{1}{5} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

$$\lim_{s\to 0} G(s) = 4 \quad (1)$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{1}{s^2}}{1+\frac{G(s)}{s}}$$

$$= \frac{1}{\lim_{s\to 0} s + \lim_{s\to 0} s\times\frac{G(s)}{s}}$$

$$= \frac{1}{0+4}$$

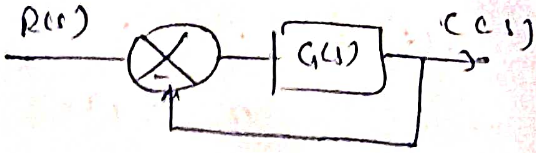
$$= \frac{1}{4} = 0.25 \text{ unit}$$

27.

let the system be

$r(t) = t$

$R(s) = \frac{1}{s^2}$



$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^2}}{1 + G(s)}$$

$$\frac{\cancel{5}}{\cancel{100}\ 20} = \frac{1}{\lim_{s \to 0} s \cdot G(s)}$$

$$\cancel{K_v} = \frac{1}{20} = \frac{1}{K_v}$$

$$\Rightarrow \boxed{K_v = 20}$$

~~#~~ → And for type "1" system only

$$\text{error} = \frac{A}{K_v}$$

⇒ ~~The~~ The system is "type 1".

Quest In a Control System if

1 = 5

2 = 25

3 = 625

4 = $(625)^2$

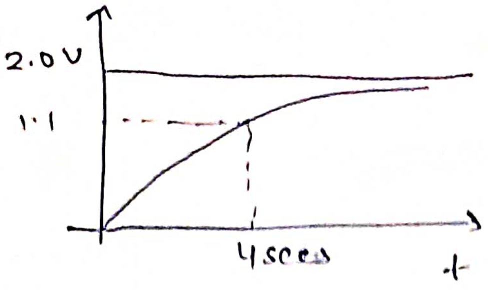
5 = 1

09/11/2008

Quest. A certain first order system is intially at rest and subjected to a sudden i/p at t = 0, its response reaches 1.1 V in 4 secs, and eventually reaches steady state of 2 V, the time constant of the system is

a) 5 secs b) 10 secs c) 15 secs d) 20 secs.

Sol:-



$$c(t) = k\left[1 - e^{-t/T}\right]$$

At $t = 4$ secs.

$$1.1 = 2\left[1 - e^{-4/T}\right]$$

$$2 \cdot e^{-4/T} = 0.9$$

$$e^{-4/T} = 0.45$$

Taking log on both sides

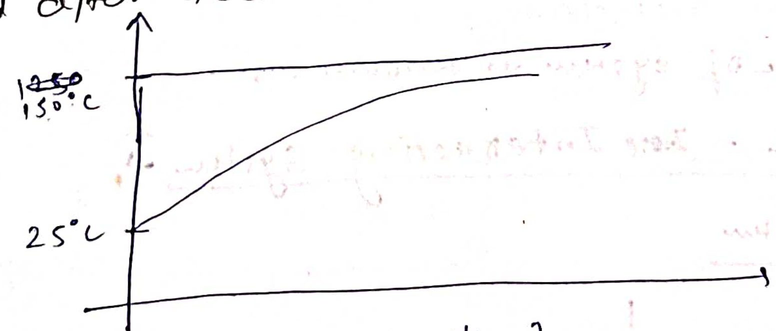
$$-\frac{4}{T} = \ln 0.45$$

$$-\frac{4}{T} = -0.79$$

$$\boxed{T = 5 \text{ secs}}$$

Ques: A thermometer having ~~a~~ 1st order dynamic is subjected to a sudden temp. change of 25°C to 150°C. If it has the time constant of 4 secs, what temp. will be indicated after 4 secs.

Sol:



$$c(t) = k\left[1 - e^{-t/T}\right]$$ → not valid, ~~for~~ if the response not starting from zero.

Imp

For response starting from non-zero value the equ. is:-

$$\boxed{c(t) = k_o + \left[k_i - k_o\right] e^{-t/T}}$$

k_i = Initial value

and k_o = Final value.

Here $k_i = 25$ and $k_o = 150$.

$C(t) = 150 + [25 - 150] e^{-t/4}$

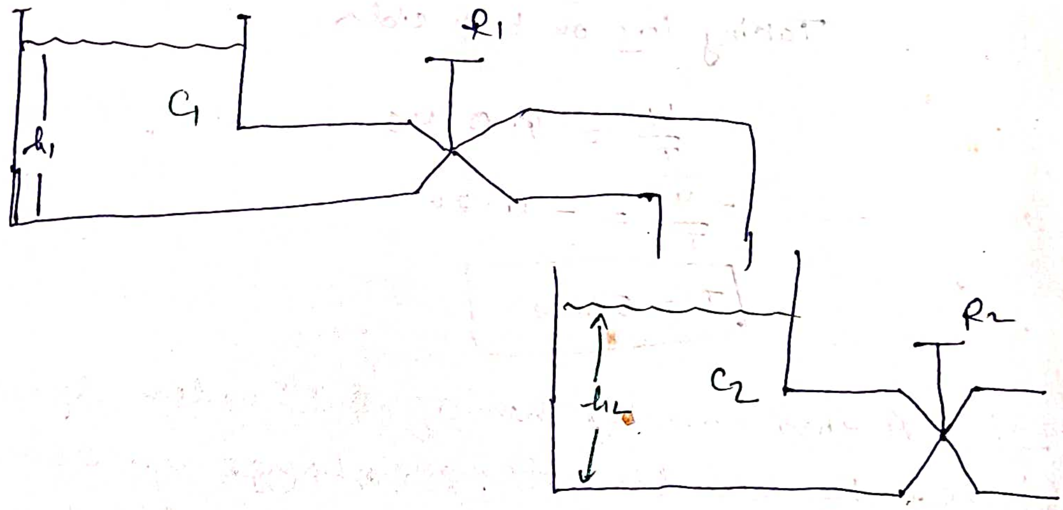
$= 150 + (-125)\, 0.37$

$= 103.75°C$.

W.B

Ex:- 3

Qus:- 3



#→ In this case any change in "h_1" will effect "h_2", but vice versa is not true.

This type of system is known as

Non - ~~Ine~~ Interacting System.

For this system

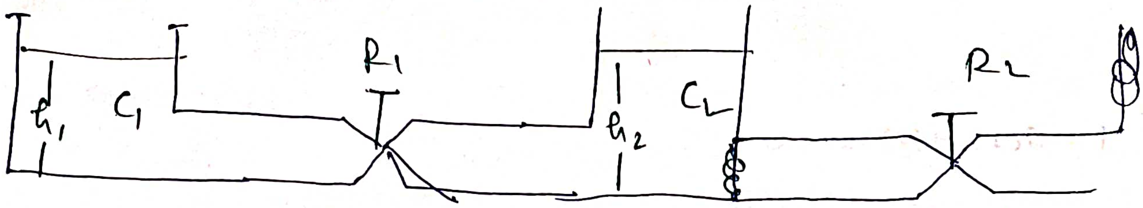
$$\frac{C(s)}{R(s)} = \frac{1}{(T_1 s + 1)(T_2 s + 1)}$$

$T_1 = T_2 = T$

$R_1 C_1 = R_2 C_2 = RC$

$$\therefore \frac{C(s)}{R(s)} = \frac{1}{(1 + Ts)^2} \rightarrow \text{(Critically damped)}$$

And when



→ In this type h_1 effect h_2 as well as h_2 effect h_1

Such system is ~~kno~~ known as

Interacting Systems I

$$\frac{C(s)}{R(s)} = \frac{1}{T^2s^2 + 3Ts + 1}$$

$T_1 = T_2 = T$ } → overdamped.

$R_1C_1 = R_2C_2 = RC$

4:=>

$$\frac{Y(s)}{X(s)} = H(s) = \frac{(s+1)}{s^2+2s+2}$$

$x(t) = e^{-t}u(t)$

$$\therefore X(s) = \frac{1}{s+1}$$

$$\therefore Y(s) = \frac{1}{\cancel{(s+1)}} \times \frac{\cancel{(s+1)}}{s^2+2s+2}$$

$$Y(s) = \frac{1}{s^2+2s+2}$$

$$\therefore Y(s) = \frac{1}{(s+1)^2 + 1^2}$$

$$\therefore \boxed{y(t) = e^{-t}\sin t}$$

6):-

$c(t) = 1 - 10e^{-t}$

$\frac{d}{dt}c(t) = 0 + 10e^{-t}$

Impulse response $= 10e^{-t}$

$\cancel{\mathcal{L}^{-1}(c(t))} = \mathcal{L}\, c'(t) = T.F = \frac{C(s)}{R(s)} = \frac{10}{(s+1)}$

11:- $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+4)} = 0$$

$$s^2 + 4s + K = 0$$

$$\omega_n = \sqrt{K}$$

$$2 \times \zeta \times \sqrt{K} = 4$$

$$\zeta = 0.5$$

$$\sqrt{K} = 4$$

$$K = 16$$

12:- $G(s) = \frac{K}{s(s+1)}$

$$e_{ss} = \frac{\lim_{s \to 0} \cancel{s} \cdot \frac{1}{s^2}}{1 + \frac{K}{s(s+1)}}$$

$$= \frac{1}{\lim_{s \to 0} s + \lim_{s \to 0} \frac{K}{s+1}}$$

$$e_{ss} = \frac{1}{K}$$

14:- $\frac{Y(s)}{U(s)} = H(s) = \frac{(s+c)}{(s+a)(s+b)}$

I/p = $u(t)$

o/p = $2 + De^{-t} + Ee^{-3t}$

$Y(s)$ for i/p $u(t)$ or $\frac{1}{s}$

$$Y(s) = \frac{(s+c)}{s(s+a)(s+b)} = \frac{A}{s} + \frac{B}{(s+a)} + \frac{C}{s+b}$$

$y(t) = 2 + De^{-t} + Ee^{-3t}$

As $t \to \infty$.

Steady state value of response $= 2$

$\therefore \Rightarrow \boxed{H = 2}$

Now for i/p $= e^{-2t}$

resp $= Fe^{-t} + Ge^{-3t}$

$H(s) = \dfrac{(s+c)}{(s+a)(s+b)}$

$H(s) = \dfrac{(s+c)}{(s+2)(s+a)(s+b)}$

$= Fe^{-t} + Ge^{-3t}$

for $\boxed{c = 2} \Rightarrow H(s) = \dfrac{1}{(s+a)(s+b)}$

15:- $M_p = 50\%$

$M_p = \dfrac{50}{100} = 0.5$

$e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.5$

$\ln e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = \ln 0.5$

$\zeta = 0.215$

Now $1 + G(s)H(s) = 0$

$1 + \dfrac{K}{s(1+s)} = 0$

$s^2 + s + K = 0$

$\omega_n = \sqrt{K}$

$2\zeta\sqrt{K} = 1$

$2 \times 0.215 \times K = 1$

$K = 5.3$

300

(17):-

$t_p = \frac{n\pi}{\omega_d}$

Here $n = 1$

$t_p = \frac{\pi}{\omega_d}$

$\omega_n = 4\ r/s$

$2 \times \zeta \times 4 = 4$

$\zeta = 0.5$

$t_p = \frac{\pi}{4\sqrt{1-(0.5)^2}}$

$t_p = \frac{\pi}{2\sqrt{3}}$ secs

(19):-

$t_s = \frac{4}{\zeta\omega_n}$

Now $\omega_n = 3$

$\therefore 2 \times \zeta \times 3 = 4$

$\zeta = \frac{2}{3}$

$\therefore t_s = \frac{\cancel{4}\,2}{\frac{\cancel{2}}{\cancel{3}} \times \cancel{3}} = 2$ secs

(21):-

$Ks^2 + s + 6 = 0$

$s^2 + \frac{1}{k}s + \frac{6}{k} = 0$

$\omega_n = \sqrt{\frac{6}{k}}$

$2 \times \zeta \times \sqrt{\frac{6}{k}} = \frac{1}{k}$

$\zeta = 0.5$ (given)

$\therefore 2 \times 0.5 \times \frac{\sqrt{6}}{\cancel{\sqrt{k}}} = \frac{1}{\cancel{k}\,\sqrt{k}}$

$\sqrt{K} = \frac{1}{2 \times 0.5 \times \sqrt{6}} \Rightarrow \boxed{K = \frac{1}{6}}$

22:-

$$G(s) = \frac{100}{(s+1)(s+100)}$$

$(s+1)(s+100) = 0$

$s^2 + 100s + s + 100 = 0$

$s^2 + 101s + 100 = 0$

negligible.

$$\therefore G(s) \approx \frac{100}{(s+1)}$$

$\Rightarrow g(t) = 100e^{-t}$

$\therefore e^{-t} \approx e^{-t/T}$

$\Rightarrow \cancel{t} \approx \frac{\cancel{t}}{T}$

$T \approx 1$ ~~secs~~ secs

For 2% T.B $= 4T$

$\therefore$ 4secs.

24:⇒

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$$

$\omega_n^2 = 4$

$\omega_n = 2$

$2 \times \zeta \times \omega_n = 4$

$\zeta = 1$

25:-

$$H(s) = \frac{1}{s+2}$$

$10u(t)$

$$H(s) = \frac{10}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$H(s) = \frac{5}{s} - \frac{5}{s+2}$$

$h(t) = 5[1 - e^{-2t}]$

$\frac{99}{100} \times 5 = 4.95$

$4.95 = 5[1 - e^{-2t}]$

$5e^{-2t} = 0.05$

$\therefore e^{-2t} = 0.01$

$-2t = \ln 0.01$

$\boxed{t = 2.3 \text{ secs}}$

26:

$$[s^2+3s+2]Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(s+2)(s+1)}$$

$$\therefore X(s) = \frac{2}{s}$$

$$\therefore Y(s) = \frac{2}{s} \times \frac{1}{(s+2)(s+1)}$$

$$\therefore Y(s) = \frac{2}{s(s+2)(s+1)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

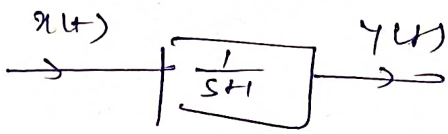
$$Y(s) = \frac{1}{s} - \frac{2}{(s+1)} + \frac{1}{(s+2)}$$

~~$y(t) = 1 - e$~~

$$y(t) = 1 - 2e^{-t} + e^{-2t}$$

28:

$$r(t) = \sin t$$



$$\frac{Y(s)}{X(s)} = F(s) = \frac{1}{(s+1)}$$

put $s = j\omega$.

$$F(j\omega) = \frac{1}{j\omega+1} = \frac{1+0j}{1+j\omega}$$

$$\therefore |F(j\omega)| = \frac{\sqrt{1^2+0^2}}{\sqrt{1^2+\omega^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$|F(j\omega)| = \frac{\tan^{-1} 0}{\tan^{-1}\omega} = -\tan^{-1}\omega$$

$r(t) = \sin t \Rightarrow \omega = 1\, r/s.$

$$F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

$$= \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\therefore Y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ).$$

30:-

$$1 + G(s)H(s) = 0$$

$$1 + \frac{(K_p + K_D s)100}{s(s+10)} = 0$$

$$s^2 + 10s + 100K_p + 100K_D s = 0$$

$$s^2 + s[10 + 100K_D] + 100K_p = 0$$

$$\omega_n = \sqrt{100K_p}$$

$$2 \times \zeta \times \sqrt{100K_p} = 10 + 100K_D$$

$$\sqrt{100K_p} = 10 + 100K_D$$

$[\zeta = 0.5]$ given

$$K_v = 1000 = \lim_{s \to 0} s \cdot G(s)H(s)$$

$$1000 = \lim_{s \to 0} \cancel{s} \times \frac{100(K_p + K_D s)}{\cancel{s}(s+10)}$$

$$10000 = 100K_p$$

$$\boxed{K_p = 100.}$$

$$\sqrt{100 \times 100} = 10 + 100K_D$$

$$100 - 10 = 100K_D$$

$$\boxed{K_D = 0.9}$$

34:-

$$\cancel{4}\frac{d^2c(t)}{dt^2} + \cancel{8}^{2}\frac{dc(t)}{dt} + \cancel{16}^{4}c(t) = \cancel{16}^{4}u(t)$$

$$\frac{d^2c(t)}{dt^2} + 2\frac{d}{dt}c(t) + 4c(t) = 4u(t)$$

$$(s^2+2s+4)\,C(s) = 4\,U(s) \quad \text{(Taking laplace)}$$

$$\frac{C(s)}{U(s)} = \frac{4}{s^2+2s+4}$$

$$\omega_n = 2\,r/s$$

$$\cancel{2}\times\zeta\,\omega_n = \cancel{2}\,1$$

$$\zeta = \frac{1}{2} = 0.5.$$

35:->

$$c(t) = \frac{1}{6}e^{-0.8t}\cdot\sin 0.6t.$$

$$T.F = \frac{C(s)}{R(s)} = \frac{\frac{1}{6}\times 0.6}{(s+0.8)^2+(0.6)^2}$$

$$(s+0.8)^2+(0.6)^2 = 0$$

$$s^2+1.6s+1 = 0$$

$$\boxed{\omega_n = 1\,r/s.}$$

$$2\zeta\times 1 = 1.6$$

$$\boxed{\zeta = \frac{1.6}{2} = 0.8}$$

37:-

$$F(s) = \frac{s}{1+s}$$

Put $s = j\omega$

$$F(j\omega) = \frac{j\omega}{1+j\omega} = \frac{0+j\omega}{1+j\omega}$$

$$|F(j\omega)| = \frac{\sqrt{0^2+\omega^2}}{\sqrt{1^2+\omega^2}} = \frac{\omega}{\sqrt{1+\omega^2}}$$

$$|F(j\omega)| = \frac{\tan^{-1}\infty}{\tan^{-1}\omega} = 90 - \tan^{-1}\omega$$

<u>Given</u>

$$x(t) = \sin t$$

$$\omega = 1\,r/s.$$

$$|F(j\omega)| = \frac{1}{\sqrt{2}}$$

$$\angle F(j\omega) = 90 - \tan^{-1}(1) = 45^\circ.$$

$$\therefore\ o/p = \frac{1}{\sqrt{2}}\sin(t+45^\circ)$$

<u>40</u>:-

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+1)} = 0$$

$$s^2+s+K = 0$$

$$\Rightarrow \omega_n = \sqrt{K}$$

$$2 \times \zeta \times \sqrt{K} = 1$$

$$\zeta = \frac{1}{2\sqrt{K}}$$

so, when $K \to \infty$

$$\boxed{\zeta = 0.}$$

<u>41</u>:-

let $\frac{Y(s)}{X(s)} = H(s) = \frac{2(s+3)}{(s+2)(s+4)}$

$$X(t) = u(t)$$

$$\Rightarrow X(s) = \frac{1}{s}$$

$$\therefore\ Y(s) = \frac{1}{s} \times \frac{2(s+3)}{(s+2)(s+4)}$$

Now ~~ss~~ Steady-state response ~~=~~ is

using final value theorem $\Rightarrow \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} \cancel{s} \times \frac{2(s+3)}{\cancel{s}(s+2)(s+4)}$

$$\frac{2(3)}{8} = \frac{\cancel{6}\,3}{\cancel{8}\,4} = \frac{3}{4}$$

42:-

$$F(s) = \frac{(s+1)}{s}$$

$$x(t) = \cos t$$

$$F(j\omega) = \frac{j\omega + 1}{0 + j\omega}$$

$$|F(j\omega)| = \frac{\sqrt{1+\omega^2}}{\omega}$$

$$\angle F(j\omega) = \tan^{-1}\omega - 90°$$

Given $\cos t$ = Input

$\Rightarrow \omega = 1\ r/s$

$\therefore |F(j\omega)| = \sqrt{2}$

$\angle F(j\omega) = 45 - 90° = -45°$.

$\therefore$ i/p = $\cos t$

$\Rightarrow$ o/p = $\sqrt{2}\cos(t - 45°)$

$= \cancel{\sqrt{2}\sin(90-t)}$

$\sqrt{2}[\cos t \cos 45° + \sin t \sin 45°]$

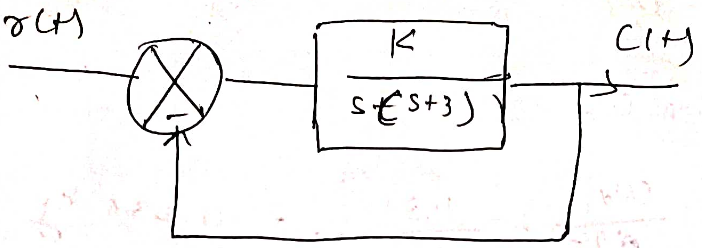
$\Rightarrow [\cos t + \sin t]$

$A\sin\omega t + B\cos\omega t = \sqrt{A^2+B^2}\cdot\sin[\omega t + \tan^{-1} B/A]$

$\sqrt{2}\sin[t + \tan^{-1}(1)]$

$\sqrt{2}\sin[t + 45°]$

Ques.



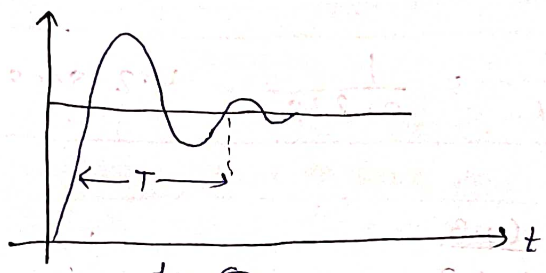


fig ①

a) what should be the min value of gain K, so that response exhibits oscillations as shown in fig ①

b) For K = 2×(min value), find the time period of oscillation shown in fig ①

Sol:
a)
$$1 + G(s)H(s) = 0$$
$$s^2 + 3s + K = 0$$
$$\omega_n = \sqrt{K}$$
$$2 \times \zeta \times \sqrt{K} = 3$$
for Kmin → $\zeta = 1$
$$\therefore \sqrt{K} = \frac{3}{2}$$
$$\boxed{K = 2.25}$$

b) Now for $K = 2 \times 2.25 = 4.5$,
$$\omega_n = \sqrt{4.5} = 2.12 \text{ r/s}$$
$$2\zeta \times 2.12 = 3$$
$$\zeta = \frac{3}{2 \times 2.12} = 0.707$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.12\sqrt{1-(0.707)^2}$$
$$\omega_d = 1.5 \text{ r/s}$$

$2\pi f_d = \omega_d$

$2\pi f_d = \omega_d$

$f_d = \frac{\omega_d}{2\pi} = \frac{1.5}{2\pi} = 0.238\ Hz$

$$\boxed{T = \frac{1}{f_d} = \frac{1}{0.238} = 4.2\ secs.}$$

W.B

Ex 1.2

(6) Sr a) $K = 32$

$s^2 + 2s + 32 = 0$

$\omega_n = \sqrt{32} = 5.65\ r/s$

$2 \times \zeta \times 5.65 = 2$

$\zeta = \frac{2}{2 \times 5.65} = 0.176.$

$\omega_{d1} = 5.65\sqrt{1-(0.176)^2} = 5.56\ r/s.$

For $k = 16$

$s^2 + 2s + 16 = 0$

$\omega_n = \sqrt{16} = 4.$

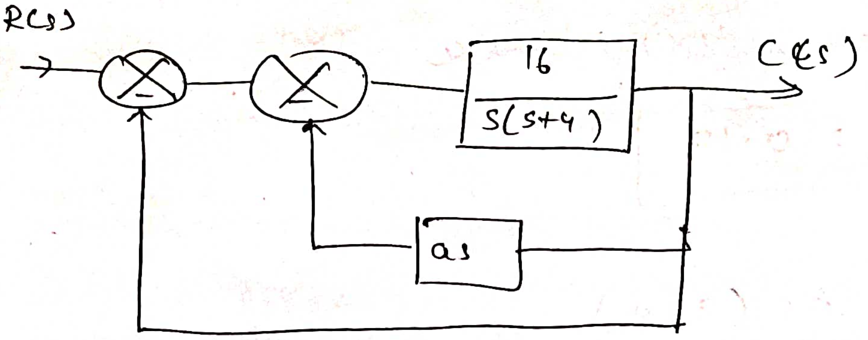
$2 \times \zeta \times 4 = 2$

$\zeta = \frac{1}{4} = 0.25.$

$\therefore\ \omega_{d2} = 0.25\sqrt{1-(0.25)^2} = 3.87\ r/s$

$$\therefore\ \boxed{\frac{\omega_{d1}}{\omega_{d2}} = \frac{5.56}{3.87} = 1.44}$$

Quest.

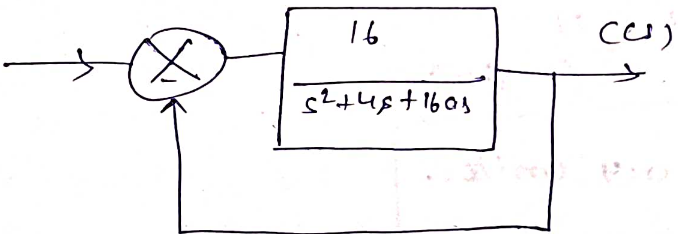


a):→ What should be the value of "a" such that $M_p = 1.5\%$. Also find steady state error for unit ramp input, with the without "a".

b):- What manipulation must be done in the system so that the steady state error for unit ramp I/P is reduces to a value obtain without "a" and keeping $M_p = 1.5\%$ at the same time.

Sol.

$$\frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}as} = \frac{16}{s^2+4s+16as}$$



(a)

$$1 + G(s)H(s) = 0$$

$$1 + \frac{16}{s^2+4s+16as} = 0$$

$$s^2+4s+16as+16 = 0$$

$$s^2+4s[1+4a]+16 = 0$$

$$\omega_n = \sqrt{16} = 4.$$

$$2 \times \zeta \times 4 = 4[1+4a]$$

Now

$M_p = 1.5\%$

$$= \frac{1.5}{100} = 0.015$$

$$e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.015$$

Taking log on both side

$$\zeta = 0.8.$$

$\therefore 2 \times 0.8 \times 4 = 4 + 16 \times a$

$$\boxed{a = 0.15}$$

i) without "a" i.e. $(a = 0)$

$$G(s)H(s) = \frac{16}{s(s+4)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{16}{s(s+4)}}$$

$$\boxed{e_{ss} = \frac{4}{16} = 0.25 \text{ unit}}$$

(ii) with 'a'

$$G(s)H(s) = \frac{16}{s(s+6.4)}$$

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{s^{\cancel{2}}}}{1 + \frac{16}{s(s+6.4)}}$$

$$\boxed{e_{ss} = \frac{6.4}{16} = 0.4 \text{ units..}}$$

(b)!.

$$e_{ss} = 0.25 = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{s^{\cancel{2}}}}{1 + \frac{K}{s(s+6.4)}}$$

$$0.25 = \frac{6.4}{K}$$

$$K = \frac{6.4}{0.25} = 25.6$$

Means, making gain $(K) = 25.6$, e_{ss} become "0.25" keeping $M_p = 1.5\%$.

Ques. Open loop transfer funcn of a unity feedback control system is

$$G(s) = \frac{k_1(s+2)}{s^3 + k_2 s^2 + 4s + 1}$$

Find the value of k_1 and k_2, so that $\zeta = 0.2$ and $\omega_n = 3 r/s$.

a) $k_1 = 7$, $k_2 = 2.8$ b) $k_1 = 2.8$, $k_2 = 7$ c) $k_1 = 8$, $k_2 = 3.8$ d) $k_1 = 7$, $k_2 = 3.8$.

Sol. $1 + G(s)H(s) = 0$

$$s^3 + k_2 s^2 + 4s + 1 + k_1 s + 2k_1 = 0$$

$$s^3 + k_2 s^2 + s[4 + k_1] + [2k_1 + 1] = 0 \quad \text{—(1)}$$

This can be factorized as

$$(s+a)(s^2 + bs + c) = 0 \quad \text{—(2)}$$

$$c = \omega_n^2 = 9.$$

$$b = 2 \times \zeta \times \omega_n = 1.2$$

$\therefore$ ~~(s+a)~~ Equn. (2) becomes

$$(s+a)(s^2 + 1.2s + 9) = 0$$

$$s^3 + 1.2s^2 + 9s + as^2 + 1.2as + 9a = 0$$

$$s^3 + s^2(1.2 + a) + s(9 + 1.2a) + 9a = 0 \quad \text{—(3)}$$

Comparing (1) and (3)

$$k_2 = 1.2 + a$$

$$4 + k_1 = 9 + 1.2a.$$

Solve it further

$$2k_1 + 1 = 9a,$$

$$a = \frac{2k_1 + 1}{9}$$

$$\therefore 4 + k_1 = 9 + 1.2 \times \frac{(2k_1 + 1)}{9}$$