—:{ PART – 4 }:—

Frequency Response Analysis :- when any system is ~~suj~~ subjected to sinsoidal i/p the o/p is also sinsoidal having different magnitude and phase angle, but same i/p frequency "ω".

#→ Frequency Response Analysis implies varying "ω" from "0" to "∞" and observing ~~co~~ corrosponding ~~varitition~~ variation in mag. and phase angle of the response

Let

$$\frac{C(s)}{R(s)} = F(s) = T.F$$

Put $s = j\omega$

$F(j\omega)$ = Sinusoidal T.F

$\approx$ Sinusoidal response.

$$F(j\omega) = |F(j\omega)| \angle F(j\omega)$$

$$|F(j\omega)| = \sqrt{(\text{Real part})^2 + (\text{Img Part})^2}$$

and $\angle F(j\omega) = \cancel{\tan^{-1}\left(\frac{\text{Re Part}}{\text{Img Part}}\right)} \tan^{-1}\left(\frac{\text{Img Part}}{\text{Real Part}}\right)$

⇒ Frequency Response Plots :-

(1) Polar ~~Pol~~ Plots :-

→ Absolute value of magnitude $|F(j\omega)|$ Vs ω,

& phase angle (degree)

(2) :- Bode plots :-

Decibel (db) value of magnitude ($20 \log |F(j\omega)|$) Vs $\log \omega$.

& phase angle (deg.)

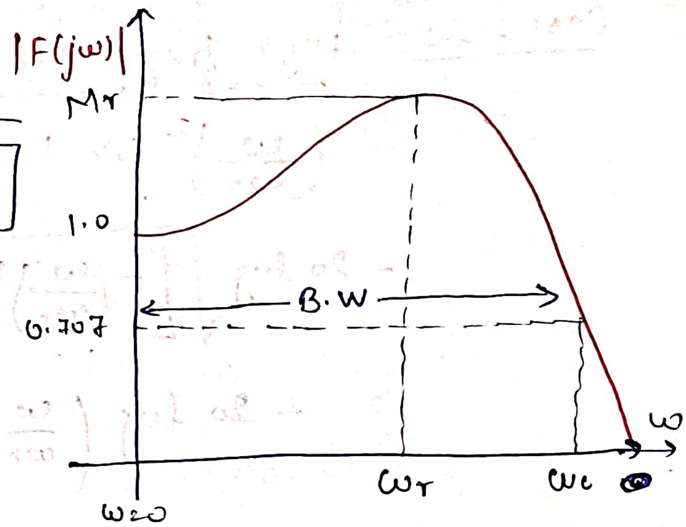
Frequency Response Analysis of 2nd order system :-

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$F(s) = \frac{1}{\left[\frac{s^2}{\omega_n^2} + \frac{2\zeta\omega_n s}{\omega_n^2} + 1\right]}$$

put $\underline{s = j\omega}$

$$F(j\omega) = \frac{1}{\left[\frac{(j\omega)^2}{\omega_n^2} + \frac{2\zeta j\omega}{\omega_n} + 1\right]}$$



$$F(j\omega) = \frac{1}{\underbrace{1-\left(\frac{\omega}{\omega_n}\right)^2}_{\text{real part}} + \underbrace{j\, 2\zeta \frac{\omega}{\omega_n}}_{\text{Img. Part}}}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

#→ For given 2nd order system :-

$$|F(j\omega)| = \left[\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}\right]^{-1}$$

Its db value is :-

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

Asymptotic Approximations :- $(\zeta = 0)$

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

Case :- 1 :- low freq. region

where $1 \gg \frac{\omega}{\omega_n}$

$$-20\log\sqrt{1} = 0 \text{ db}.$$

Case :- 2 : High freq. region :

$$\left(\frac{\omega}{\omega_n}\right)^2 >> 1$$

$$-20 \log \sqrt{\left[-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

$$= -20 \log \left(\frac{\omega}{\omega_n}\right)^2$$

$$= -40 \log \frac{\omega}{\omega_n} = -40 \log \omega + 40 \log \omega_n$$

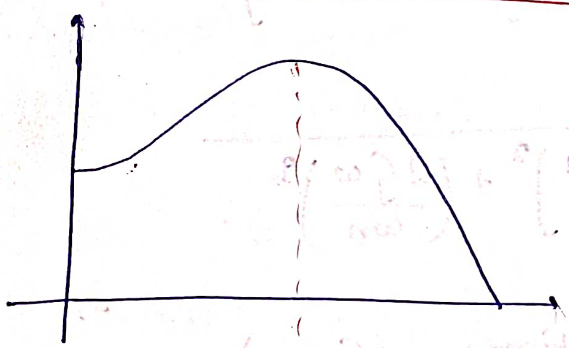
$$y = mx + c$$

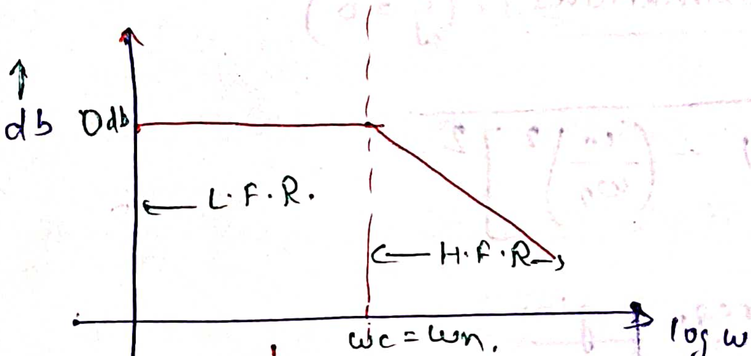
Corner freq

$$0 = -40 \log \left(\frac{\omega}{\omega_n}\right)$$

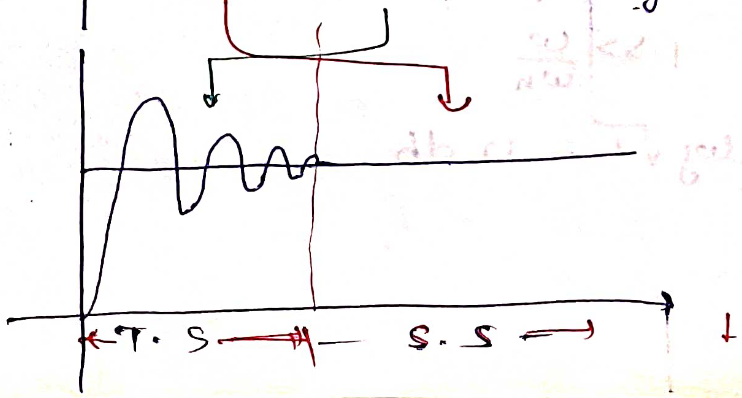
$$\log \left(\frac{\omega}{\omega_n}\right) = 0 \Rightarrow \frac{\omega}{\omega_n} = \text{Antilog}(0) = 1$$

$$\Rightarrow \boxed{\omega = \omega_{corner} = \omega_n}$$





wc → wcorner



i) **Resonant freq:- (ω_r)** :. It is defined as freq. at which magnitude has max value.

$$|F(j\omega)| = \frac{1}{\sqrt{[1-u^2]^2 + (2\xi u)^2}}$$

let $u = \frac{\omega}{\omega_n}$.

At $\omega = \omega_r$

$$u = u_r = \frac{\omega_r}{\omega_n}$$

$\Rightarrow$ To getting max $|F(j\omega)|$

$$\frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-1/2} = 0$$

$$\frac{-1}{2}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-3/2} \cdot \frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right] = 0$$

$\Rightarrow \quad 2(1-u_r^2)(-2u_r) + 4\xi^2 \cdot 2u_r = 0$

$\Rightarrow \quad (2 - 2u_r^2)(-2u_r) + 8\xi^2 u_r = 0$

$\Rightarrow \quad -4u_r + 4u_r^3 + 8\xi^2 u_r = 0$

$\Rightarrow \quad 4u_r[-1 + u_r^2 + 2\xi^2] = 0$

$\Rightarrow \quad u_r^2 = 1 - 2\xi^2$

$\Rightarrow \quad u_r = \sqrt{1-2\xi^2}$

$\Rightarrow \quad \frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}$

$\Rightarrow \quad \boxed{\omega_r = \omega_n\sqrt{1-2\xi^2} \ r/s}$

as we know

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2} \ r/s}$$

$\rightarrow$ It is correlated with "damped Natural freq" (ω_d).

(ii) Resonant Peak :- (M_r)

$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

At $\omega = \omega_r = \omega_n\sqrt{1-2\zeta^2}$:-

$|F(j\omega)| = M_r$.

$$\therefore \Rightarrow M_r = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_n\sqrt{1-2\zeta^2}}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega_n\sqrt{1-2\zeta^2}}{\omega_n}\right)^2}}$$

By simplifying this

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

- It is correlated with "M_p" i.e "Max peak Overshoot"

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

→ Both are Max. Value

→ Both are functions of "ζ" only.

(iii) Band width :- It is the range of frequencies over which the magnitude has a value of "$\frac{1}{\sqrt{2}}$"

→ ~~wider~~ $\boxed{\text{wider B.W} \rightarrow \text{Faster response}}$

→ It indicates the speed of response of the system.

$$\Rightarrow \boxed{B.W \propto \frac{1}{t_r}}$$ where (t_r → Rise time)

~~06/11/2008~~ 13/11/2008

(iv) Cut off frequency :⇒ It is defined as the freq. at which the ~~mg~~ magnitude has a value of $\frac{1}{\sqrt{2}}$.

→ It indicates the ~~adi~~ ability of the system to distinguish signal from noise.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\zeta u)^2}} \quad \left(\text{where } u = \frac{\omega}{\omega n}\right)$$

At $\boxed{\omega = \omega_c = \text{cut off freq}}$

$$u = u_c = \frac{\omega_c}{\omega n}$$

$$\frac{1}{\sqrt{(1-u_c^2)^2+(2\zeta u_c)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (1-u_c^2)^2 + (2\zeta u_c)^2 = 2$$

$$\Rightarrow u_c^4 + 1 - 2u_c^2 + 4\zeta^2 u_c^2 - 2 = 0$$

$$\Rightarrow u_c^4 + u_c^2[4\zeta^2 - 2] - 1 = 0$$

Now let $\boxed{y = u_c^2}$

$$y^2 + y[4\zeta^2 - 2] - 1 = 0$$

∴ root of the given equ. is

$$\frac{-[4\zeta^2 - 2] \pm \sqrt{(4\zeta^2-2)^2+4}}{2}$$

$$\Rightarrow [1-2\zeta^2] \pm \frac{\sqrt{16\zeta^4 - 16\zeta^2 + 4 + 4}}{2}$$

$$\Rightarrow [1-2\zeta^2] \pm \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$ (as freq. can't ~~negative~~ negative)

$$\therefore y = 1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$u_c = \sqrt{1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\therefore \boxed{\omega_c \text{ or } B.W = \omega_n\sqrt{1-2\zeta^2+\sqrt{4\zeta^4-4\zeta^2+2}}}$$

Band width.

Imp. Point :- $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

For freq. ω_r to be real.

$$1-2\zeta^2 > 0$$

$$\Rightarrow 2\zeta^2 < 1$$

$$\Rightarrow \zeta^2 < 1/2$$

$$\Rightarrow \zeta < \frac{1}{\sqrt{2}} = 0.707$$

$\therefore$ when

$$\zeta < \frac{1}{\sqrt{2}} \Rightarrow M_r > 1$$

$$\zeta = \frac{1}{\sqrt{2}} \Rightarrow M_r = 1$$

$$\zeta > \frac{1}{\sqrt{2}} \Rightarrow \text{No } M_r.$$

as, the ω_r itself become img.

Ques:- A IInd order CS has

$$M(j\omega) = \frac{100}{100 - \omega^2 + 10\sqrt{2}j\omega}$$

its peak magnitude (M_p) is

a) 0.5   b) 1   c) $\sqrt{2}$ d) 2

Sol:- $$M(j\omega) = \frac{1}{1 - \frac{\omega^2}{100} + j\frac{\cancel{10}\sqrt{2}\,\omega}{\cancel{100}\,10}}$$

$$= \frac{1}{1 - \frac{\omega^2}{100} + j\frac{\sqrt{2}}{10}\omega} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\,2\zeta\frac{\omega}{\omega_n}}$$

$$2\zeta \frac{\cancel{\omega}}{\omega_n} = \frac{\sqrt{2}\,\cancel{\omega}}{1\cancel{0}}$$

$$\zeta = \frac{\sqrt{2}}{2} = 0.7 \Rightarrow \boxed{M_p > 1}$$

W.B

Ex:6

20) :-

~~$s^2+10s+1$~~ $G(s) = \dfrac{100}{s}$

$$\omega_n^2 = 100$$

$$\omega_n = 10$$

$$2\times\zeta\times\omega_n = 10$$

$$2\times\zeta\times \cancel{10} = \cancel{10}$$

$$\zeta = \frac{1}{2} = 0.5$$

→ wrong method

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2\times0.5\sqrt{1-0.5^2}} = \frac{1}{\sqrt{1-0.5^2}}$$

$$= 1.15$$

$$\omega_r = \cancel{100}\,\omega_n\sqrt{1-2\zeta^2}$$

$$= 10\sqrt{1-2\times(0.5)^2}$$

$$\omega_r = 7.07 \text{ rad/s}$$

Stability from frequency Response Plots:-

$$1 + G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = -1$$

$$\Rightarrow \cancel{G}\, G(j\omega)H(j\omega) = -1 + j0$$

[critical point]

1):- Gain crossover frequency:-
(ω_{gc})

$$|\cancel{G}\, G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

(2) :- **Phase crossover frequency :-**
(ω_{pc})

$$\angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{pc}} = \pm 180^\circ .$$

(3) :- **Gain Margin (G.M) :-**

→ It is the "allowable gain".

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = x$$

$$G.M = \frac{1}{x} \quad ; \quad G.M\,(dB) = 20\log(1/x)$$

→ G.M is always Expressed in (dB).

(4) :- **Phase Margin (P.M) :**

→ It is the "allowable ~~Phag~~ Phase lag".

$$\angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{gc}} = \phi$$

→ $\boxed{P.M = 180^\circ + \phi}$

$$\left\{ \begin{array}{l} \text{# → Stable} \Rightarrow G.M = P.M = +ve \\ \text{Unstable} \Rightarrow \;\;"\;\; = \;\;"\;\; = -ve . \\ \text{Marginally stable} \Rightarrow \;\;"\;\; = \;\;"\;\; = 0 , \end{array} \right\}$$

W.B
Ex :- 6
3.6 :-

$$G(s) = \frac{2(\cancel{1}+s)}{s^2}$$

$$G(j\omega) = \frac{2(1+j\omega)}{(j\omega)^2} = \frac{(2+j0)(1+j\omega)}{(0+j\omega)(0+j\omega)}$$

$$\angle G(j\omega) = \frac{[0^\circ][\tan^{-1}\omega]}{[90^\circ][90^\circ]}$$

$$\angle G(j\omega) = -180^\circ + \tan^{-1}\omega$$

At $\omega = \omega_{PC} = \infty$ r/s.

$$|G(j\omega)| = \frac{2\sqrt{1+\omega^2}}{\omega^2}$$

$\Rightarrow x = |G(j\omega)|_{\omega = \omega_{pc}} = \infty$

$\therefore$ $G.M = \frac{1}{x} = \frac{1}{\infty} = 0$

$G.M(db) = 20\log 0 = 20\log\frac{1}{\infty} = 20\log 1 - 20\log\infty$

$= -\infty$.

Hence :- (d).

$\cancel{\cos\phi + j\sin}$

$\cos\phi + j\sin\phi$

(47) :-

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3\times 1}{\omega\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = \frac{[0][-57.3\times 2\times\omega]}{[90]\left[\tan^{-1}\frac{\omega}{2}\right]}$$

$$\angle G(j\omega) = -114.6\omega - 90^\circ - \tan^{-1}\frac{\omega}{2}$$

At $\omega = 0.5 \Rightarrow \angle G(j\omega) = -178^\circ$

at $\omega = 0.65 \Rightarrow \angle G(j\omega) = -182^\circ \approx -180^\circ$

$\omega_{pc} = 0.65$ r/s.

$$|G(j\omega)|_{\omega = \omega_{pc} = 0.65\text{ r/s}} = X = \frac{3}{0.65\times\sqrt{0.65^2+4}}$$

$\therefore$ $G.M = \frac{1}{X} = \frac{1}{2.2} = 0.45$

$$\boxed{G.M\ (db) = 20 \log 0.45 = -7\ db.}$$

$$|G(j\omega)| = \frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$3 = \omega\sqrt{\omega^2+4}$$

$$9 = \omega^2(\omega^2+4)$$

$$\Rightarrow \omega^4 + 4\omega^2 = 9$$

$$\omega^4 + 4\omega^2 - 9 = 0$$

put $y = \omega^2$

$$\Rightarrow y^2 + 4y - 9 = 0$$

$$y = \frac{-4 \pm \sqrt{4^2 + 36}}{2}$$

$$= \frac{-2 \pm \sqrt{4+36}}{2} \quad -2 \pm \sqrt{52}$$

$$-2 \pm \sqrt{40} \quad -2 \pm 3.6$$

$$-2 \pm 6$$

$$\therefore y = 1.6, -5.6$$

$$\therefore \omega^2 = 1.6$$

$$\Rightarrow \omega = \omega_{gc} = \sqrt{1.6} = 1.26\ r/s$$

At $\omega = \omega_{gc} = 1.26\ r/s$

$$\angle G(j\omega) = \phi$$

$$\phi = -114.6 \times 1.26 - 90 - \tan^{-1}\frac{1.26}{2}$$

$$\phi = -267^\circ$$

$$\therefore P.M = 180 + \phi$$

$$= -87^\circ,$$

(45) ! ⇒

$$G(s) = \frac{as+1}{s^2}$$

$$G(j\omega) = \frac{a(j\omega)+1}{(j\omega)^2}$$

Given $P.M = 45°$.

$P.M = 180° + \phi$

$45 = 180° + \phi$

$\phi = -135°$.

$$\angle G(j\omega) = \frac{\tan^{-1}\omega a}{180°}$$

$$= -180° + \tan^{-1}\omega a$$

If $\omega = \omega_{gc}$

$\angle G(j\omega) = \phi$

$\Rightarrow -180° + \tan^{-1}\omega a = -135°$

$\tan^{-1}\omega a = 45°$

$\Rightarrow \omega a = \tan 45° = 1$

$\Rightarrow$ $\boxed{\omega = \omega_{gc} = \frac{1}{a}}$

Now $$|G(j\omega)| = \frac{\sqrt{1+(\omega a)^2}}{\omega^2}\Bigg|_{\omega=\omega_{gc}=\frac{1}{a}} = 1$$

$$\frac{\sqrt{1+\left(\frac{1}{a}\times a\right)^2}}{\frac{1}{a^2}} = 1 \Rightarrow \frac{1}{a^2} = \sqrt{2}$$

$$\Rightarrow a^2 = \frac{1}{\sqrt{2}} \Rightarrow a = \frac{1}{\sqrt{\sqrt{2}}} = \boxed{a = 0.84}$$

$$\therefore G(s) = \frac{0.84s+1}{s^2}$$

$$\therefore G(s) = \frac{0.84}{s} + \frac{1}{s^2}$$

$L^{-1}[G(s)] = 0.84 + \cancel{0.84} +$

$= 0.84 + 1 = 1.84$ Ans.

29.

$$G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$G(j\omega)H(j\omega) = \frac{2\sqrt{3}}{j\omega(j\omega+1)}$$

$$|G(j\omega)| = \frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}}$$

$\therefore$ for Gain crossover freq,

$$|G(j\omega)| = 1$$

$$\Rightarrow \frac{2\sqrt{3}}{\omega(\sqrt{1+\omega^2})} = 1$$

$$\Rightarrow 2\sqrt{3} = \omega\sqrt{1+\omega^2}$$

$$\omega^2(1+\omega^2) = 4\times 3$$

$$\omega^4 + \omega^2 \cancel{4} - 12 = 0$$

put $\omega^2 = y$

$$\Rightarrow y^2 + y - 12 = 0$$

$$\therefore y = \frac{-1 \pm \sqrt{1-48}}{2}$$

$$= -0.5 \pm 3.5.$$

$\therefore$ $y = 3, -4.$

$\therefore$ $\omega^2 = 3$

$$\omega = \omega_{gc} = \sqrt{3} \text{ r/s}.$$

Now $$\angle G(j\omega) = \frac{[0°]}{[90][\tan^{-1}\omega]} = -90° - \tan^{-1}\omega$$

$$-90° - \tan^{-1}\omega \Big|_{\omega=\sqrt{3}}$$

$$-90° - \tan^{-1}\sqrt{3} = -90 - 60° = -150°.$$

$\therefore \phi = -150^\circ$.

$\therefore \boxed{PM = 180 - 150^\circ = +30^\circ}$

(28) :-

$$G(s)H(s) = \frac{\sqrt{2}}{s(s+1)}$$

$$G(j\omega)H(j\omega) = \frac{\sqrt{2}}{j\omega(j\omega+1)}$$

$$\therefore \angle G(j\omega) = \frac{[0]}{[90][\tan^{-1}\omega]}$$

$$= -90^\circ - \tan^{-1}\omega.$$

Now at $\omega = \omega_{pc} = \infty$

$$\angle G(j\omega) = -90 - 90 = -180^\circ.$$

$$\therefore |G(j\omega)|_{\omega = \omega_{pc} = \infty} = X = 0$$

$$\therefore G.M = \frac{1}{X} = \frac{1}{0} = \infty.$$

58:-

$$G(s) = \frac{e^{-Ts}}{[s(s+1)]}$$

At $\omega = \omega_1$

$$\angle G(j\omega) = 0$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(1+j\omega)}$$

$$\angle G(j\omega) = \frac{-\omega T}{90^\circ[\tan^{-1}\omega]}$$

$$= -90 - \omega T - \tan^{-1}\omega$$

at $\omega = \omega_1$

$$-90 - \omega_1 T - \tan^{-1}\omega_1 = 0$$

$$-\tan^{-1}\omega_1 = 90 + \omega_1 T$$

$$\omega_1 = \tan[90 + \omega_1 T]$$

γ $\omega_1 \approx$ Cot $[\omega_1 T]$

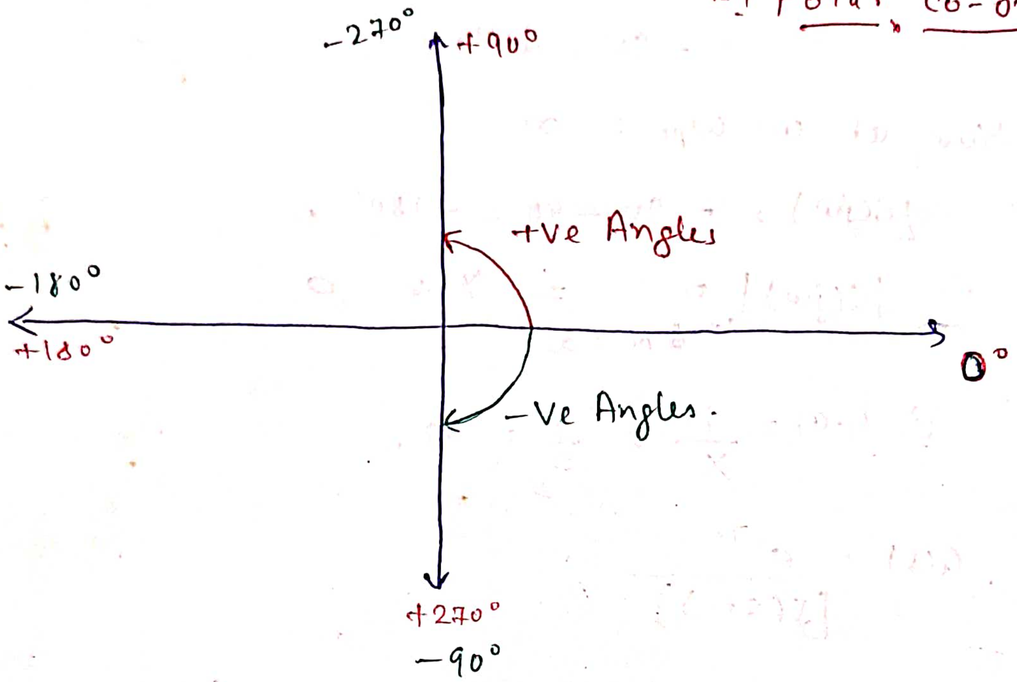
$$\boxed{\omega_1 = \text{Cot}\,[\omega_1 T]}$$

Polar Plots :- It is a plot the absolute value values of magnitude and phase angle in degrees of open loop transfer funcⁿ.

"$G(j\omega)H(j\omega)$" Vs "ω".

drawn on ~~pto~~ polar" co-ordinates:

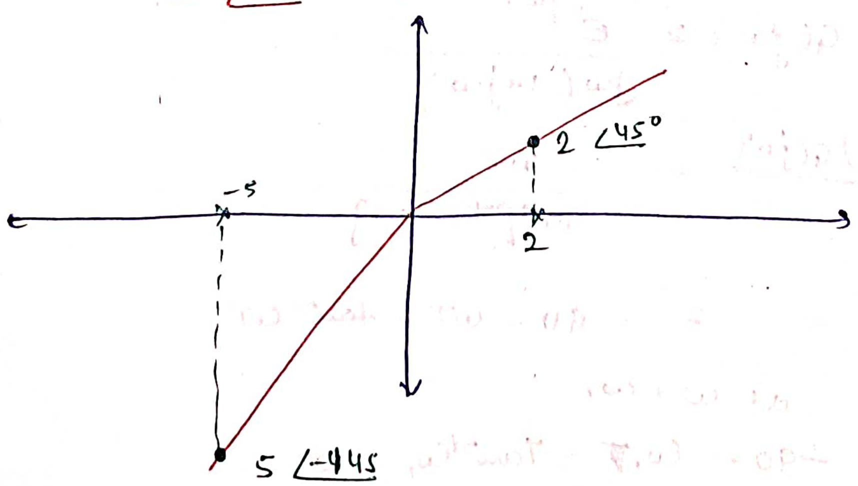
-: Polar co-ordinates :-



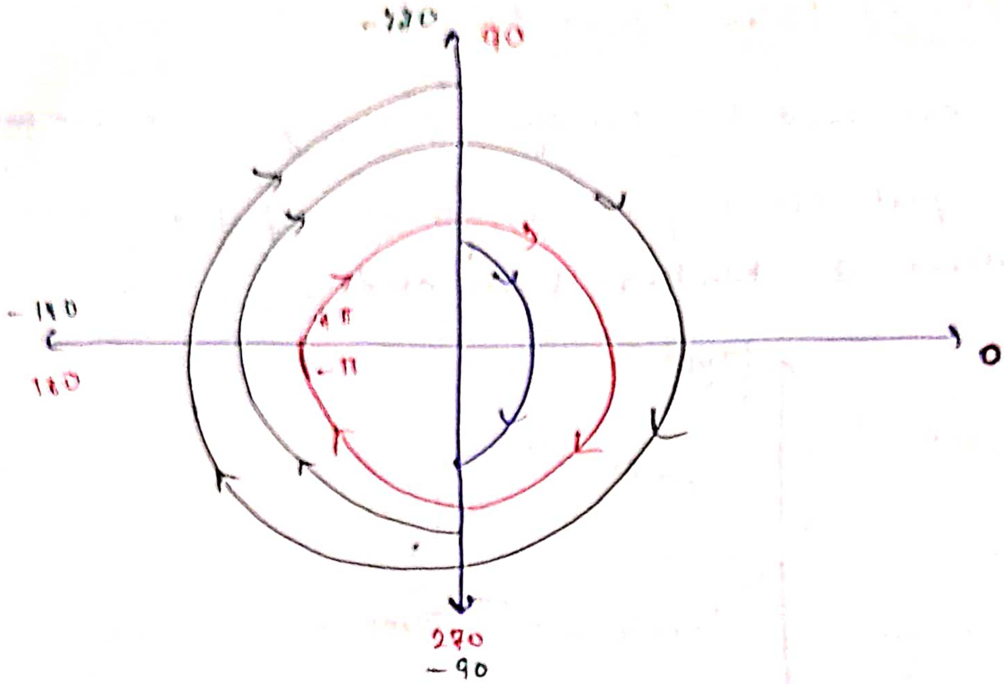
Ex:-

$5 \angle -45°$

$2 \angle 45°$



For Angles



→ +90 to −90°.

→ +180° to −180

→ +270 to −270°

$$G(S) = \frac{1}{S+1}$$

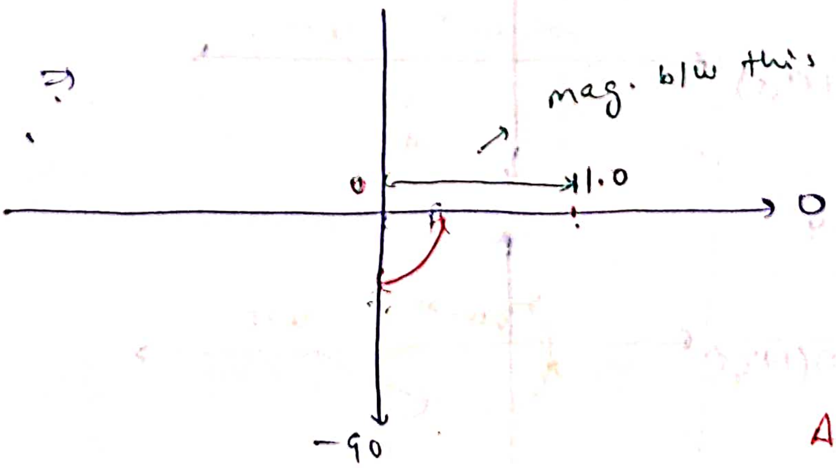
$$G(j\omega) = \frac{1}{j\omega+1}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} \quad \text{and} \quad \angle G(j\omega) = -\tan^{-1}\omega.$$

ω	0	- - - - - -	∞
$\lvert G(j\omega)\rvert$	1		0
$\angle G(j\omega)$	0°		−90°.

⇒

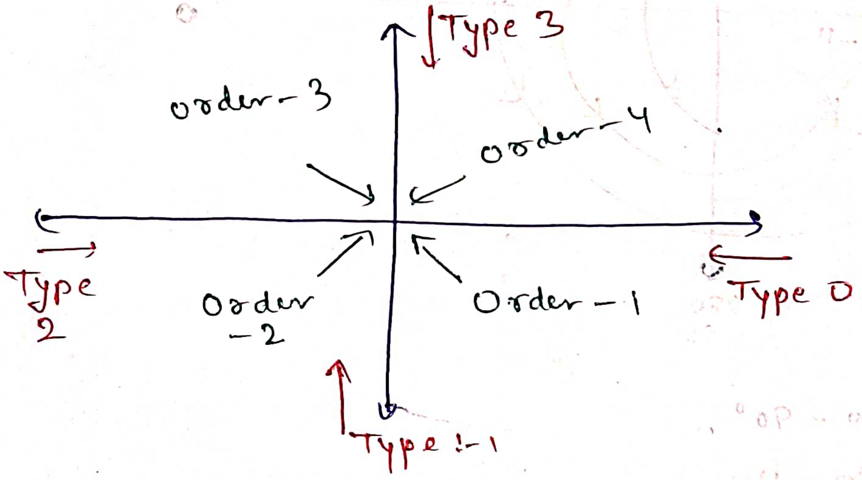
mag. b/w this range

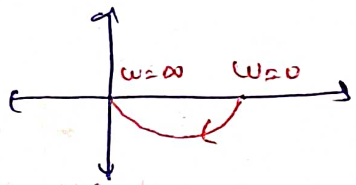
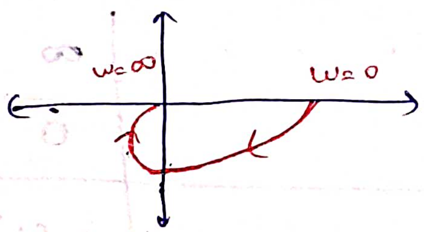
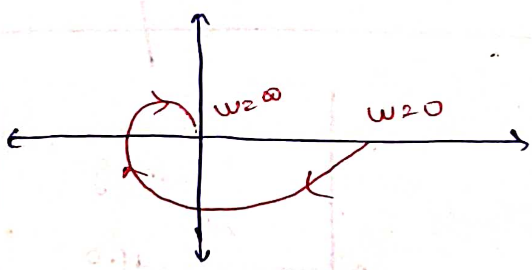
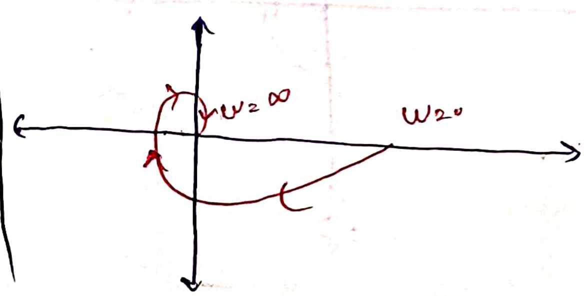


Angles b/w them

General Shapes of Polar Plots :→

#→ For certain standard T.F the type of the system determines where the polar plot starts, and the order of the system determines where the ~~plo~~ polar plots ends.

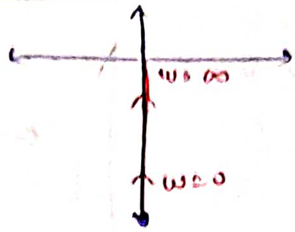


Type / order	Polar plots.
i) Type-0 / order-1 $G(s) = \frac{1}{(1+Ts)}$	
ii) Type0 / order-2 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)}$	
iii) Type-0 / order-3 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$	
iv) Type 0 / order-4 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$	

Type / order	Polar Plots

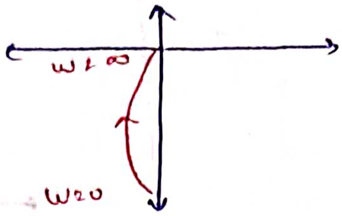
(v) Type-1 / order-1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$$



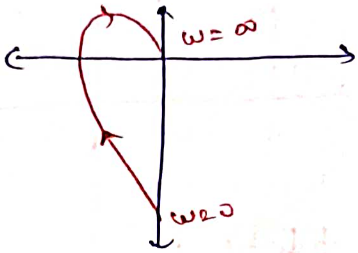
(vi) Type-1 / order-2

$$G(s) = \frac{1}{s(1+Ts)}$$



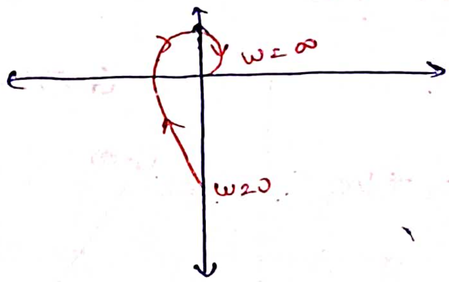
(vii) Type-1 / order-3

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)}$$



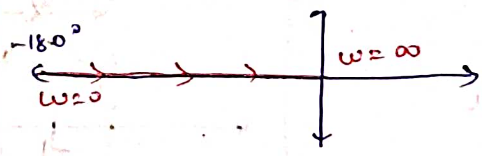
(viii) Type-1 / order-4

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$



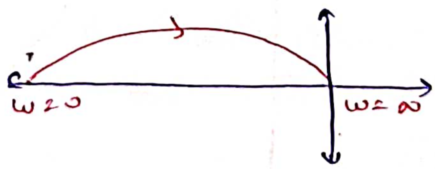
(ix) Type-2 / order-2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2}\angle -180^\circ$$



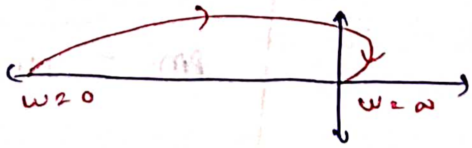
(x) Type-2 / order-3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



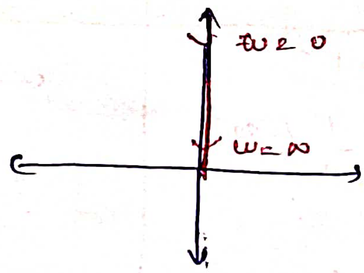
(xi) Type-2 / order-4

$$G(s) = \frac{1}{s^2(1+T_1 s)(1+T_2 s)}$$



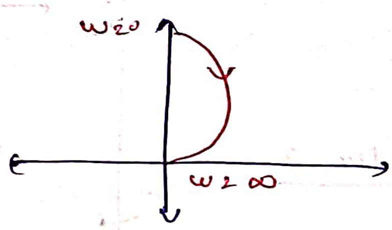
(xii) Type – 3 / order 3

$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} \angle -270^\circ$



(xiii) Type – 3 / order 4

$G(s) = \frac{1}{s^3(1+Ts)}$

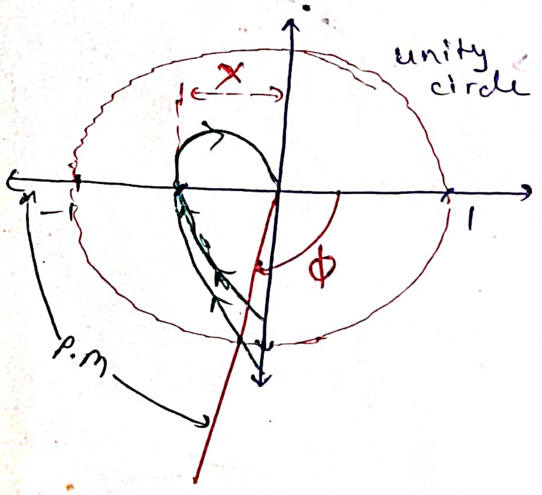


Stability from polar plots :

$G.M = \frac{1}{x}$

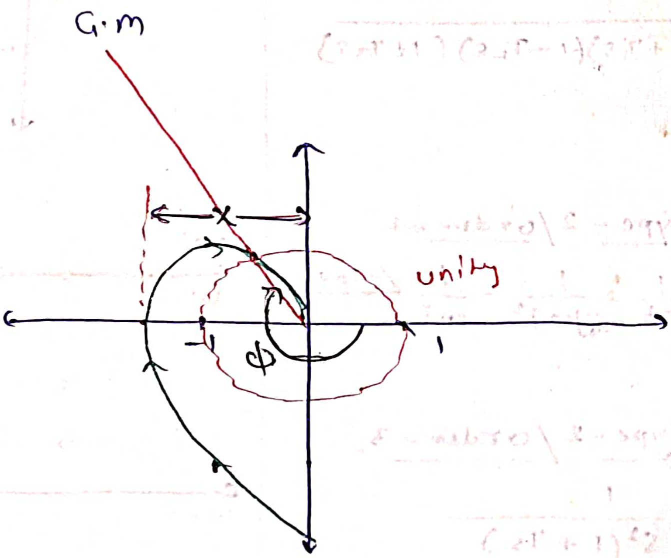
$G.M\,(dB) = 20 \log \frac{1}{x} = +ve$

$P.M = 180^\circ + \phi = +ve$.



Absolutely Stable

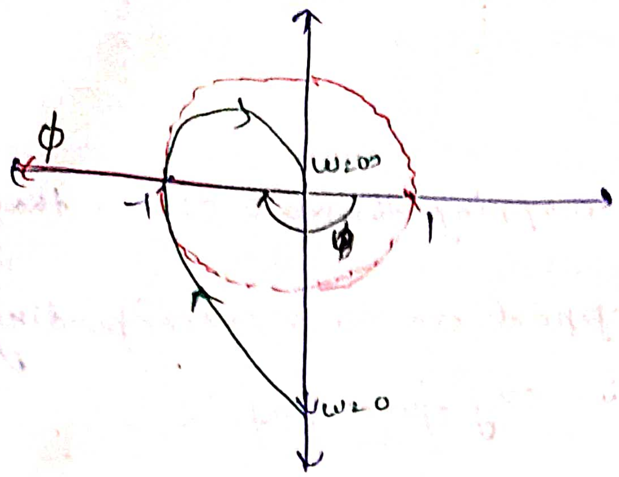
$G.M = \frac{1}{x}$



$G.M\,(dB) = 20 \log \frac{1}{x} = -ve$

$PM = 180^\circ + \phi = -ve$

Unstable



$GM(db) = 20\log 1 = 0\,db$

$\phi = -180°$

$PM = 180 - 180° = 0$

$\therefore$ System is marginally stable

Concepts of Enclosement and Encirclement :

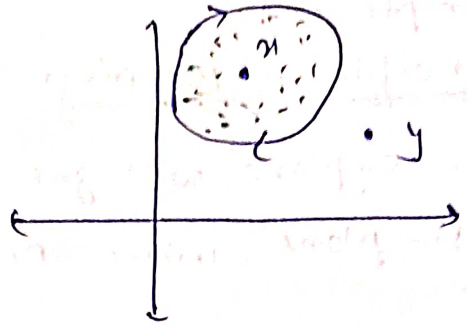


fig ①

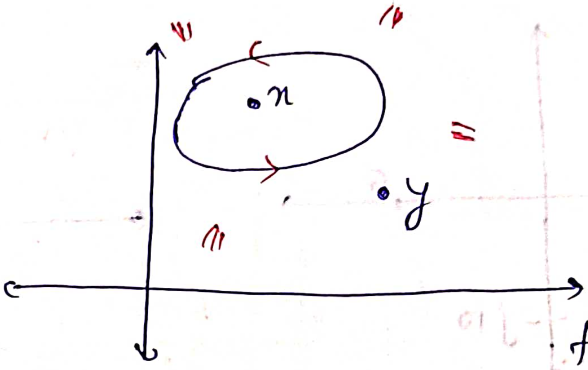


fig ②

Important Notes

#→ A pt. is said to be enclosed by a contour, if that pt. lies to the right side of the direction of contour.

#→ A pt is said to be encircled if the contour is closed path. In fig ① "x" is said to enclosed as well as encircled in clockwise direction.

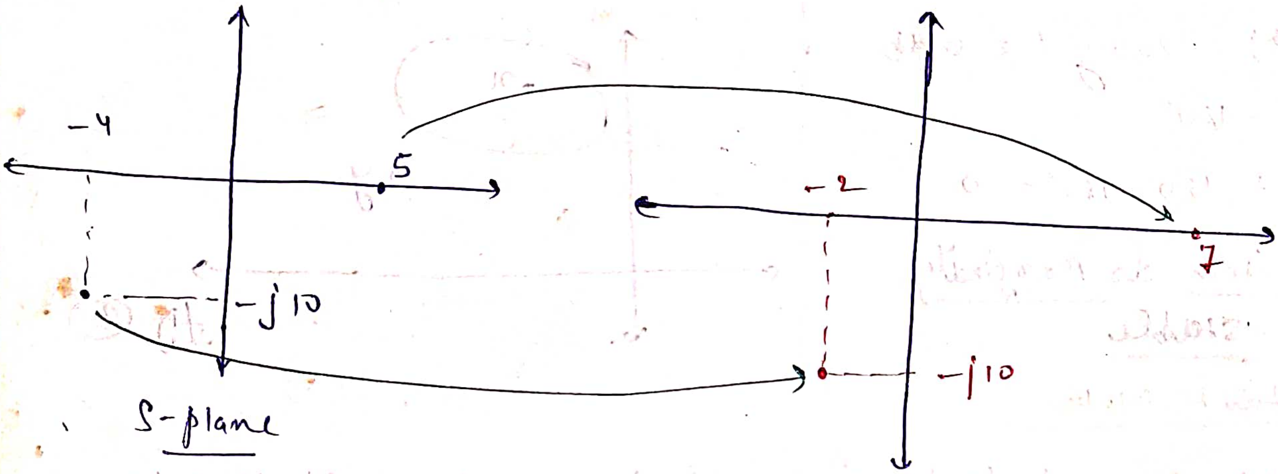
#→ In fig ② "y" is said to enclosed where as pt. "x" is said to encircled in ~~clo~~ anitclockwise direction.

#→ In polar ~~plott~~ plots, if the critical pt. $(-1+j0)$ is not enclosed, then "G.M = P.M = +ve". and the system is said to be absolutely stable

Theory of Nyquist Plots :-

#→ ~~Priciple~~ :

① **Principle of mapping:-** The mapping theorem states that every pt in "s-plane" will get mapped on to a corresponding pt. in "P(s)-plane", where P(s) is any func. of "s".



P(s)-plane

let $P(s) = s+2$

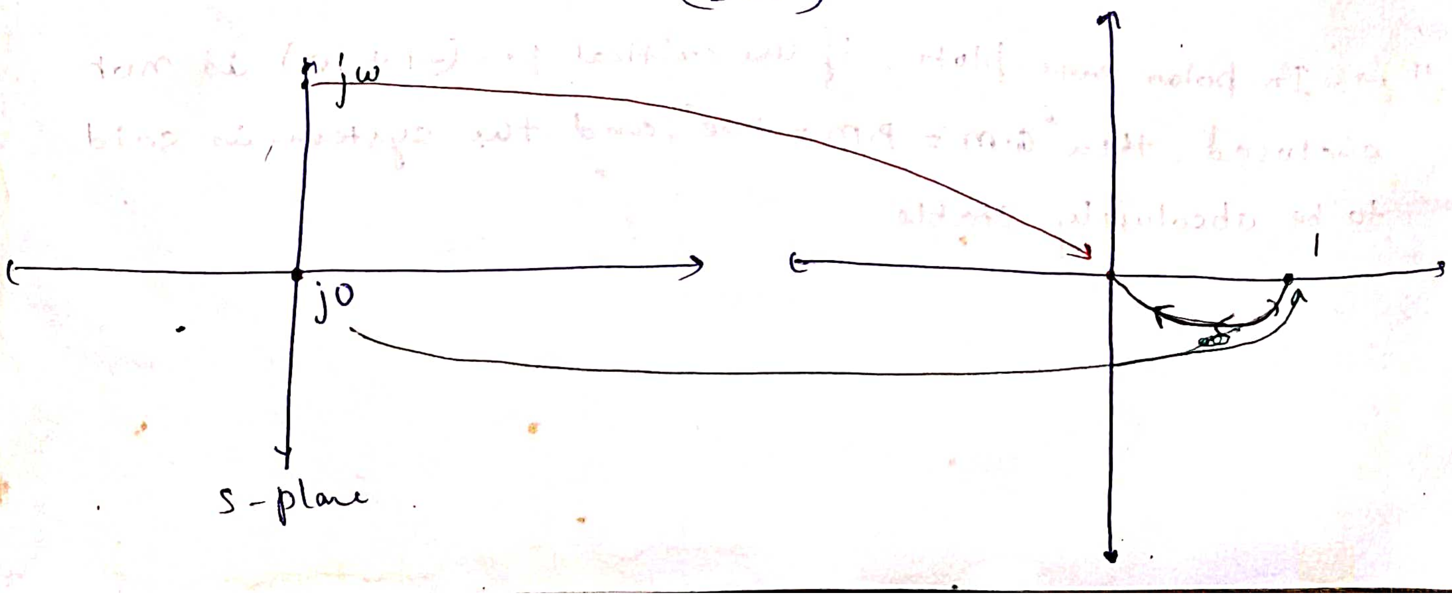
$P(5) = 5+2 = 7$

$P(-4-j10) = -4-j10+2$

$= -2-j10$

② **Polar plot :-**

Ex:- $G(s)H(s) = \dfrac{1}{(s+1)}$

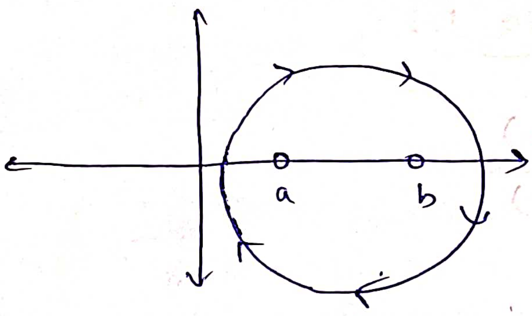


$$P(s) = G(s)H(s) = \frac{1}{-s+1}$$

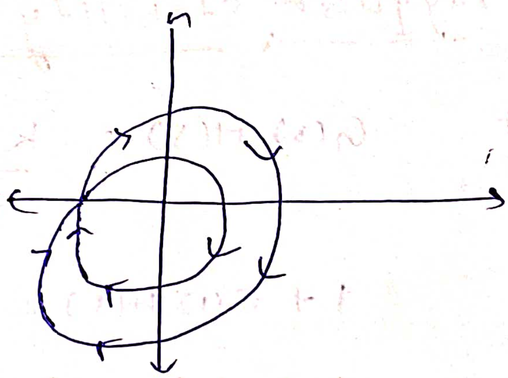
$$= \frac{1}{\sqrt{\omega^2+1}} \angle \tan^{-1}\omega$$

②:- Principle of Argument :-

Case :- 1 :- S-plane

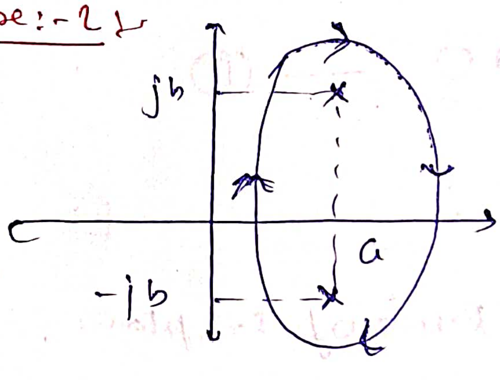


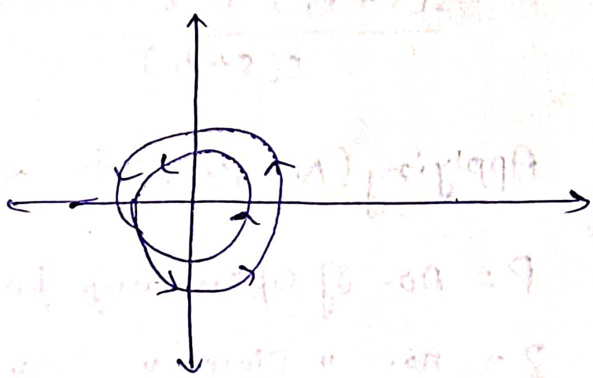
P(s) - plane



$P(s) = (s-a)(s-b)$

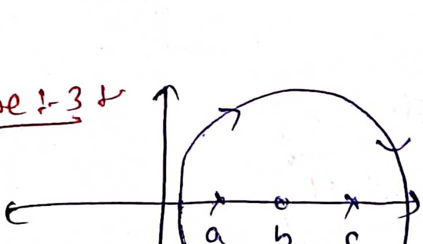
Case :- 2 :-

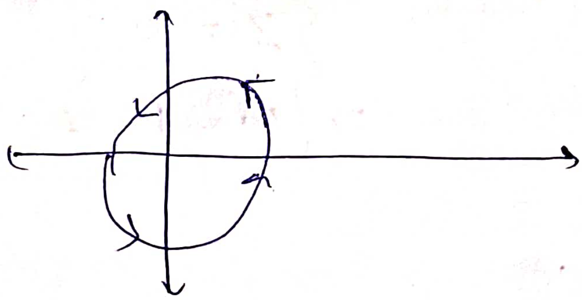




$$P(s) = \frac{1}{(s-a+jb)(s-a-jb)}$$

Case :- 3 :-





$$\boxed{N = P - Z}$$

N = No. of encirclements.

= +ve C.C.W direction (Anti Clockwise)

= −ve C.W direction

P = no. of poles in R.H.S of S-plane

Z = " " zeros " " " " "

③:- Nyquist Stability Criteria :-

let $G(s)H(s) = \dfrac{K(s+z_1)}{s(s+p_1)}$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+z_1)}{s(s+p_1)} = 0$$

$$\Rightarrow \frac{s(s+p_1) + K(s+z_1)}{s(s+p_1)} = 0 \quad \text{—— ①}$$

Applying ($N = P - Z$) to equ. ①.

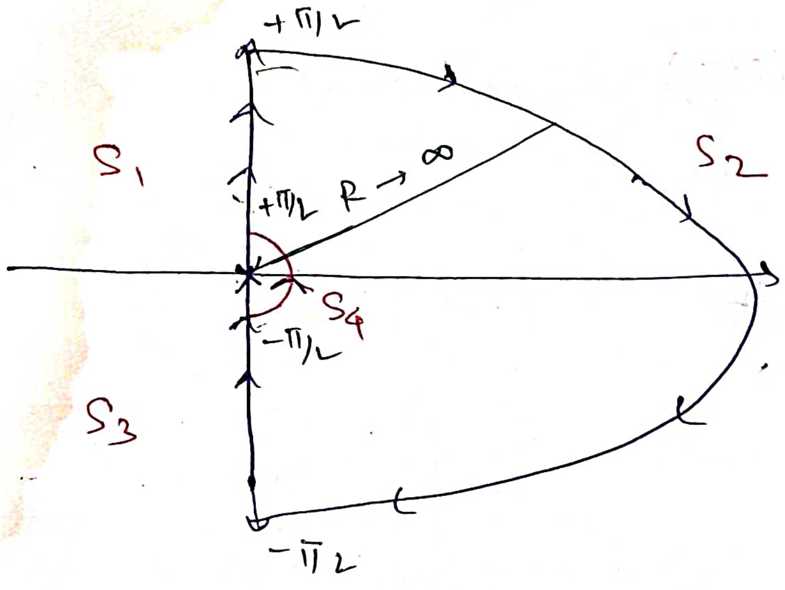
P = no. of open loop poles in R.H.S of S-plane

Z = no. " Close " " " " " " "

Imp → for system to be stable :-

$$\Rightarrow \left\{ \begin{array}{l} Z = 0 \\ N = P \end{array} \right\}$$ → Condition of Stability

Procedure for Drawing Nyquist plot :-



S-plane

#→ → To Map S_1

Polar plot

#→ → To Map S_2

Put $s = \lim_{R \to \infty} Re^{j\theta}$ (where $\theta = +\pi/2 \to -\pi/2$)

#→ → To Map S_3

Inverse Polar plot

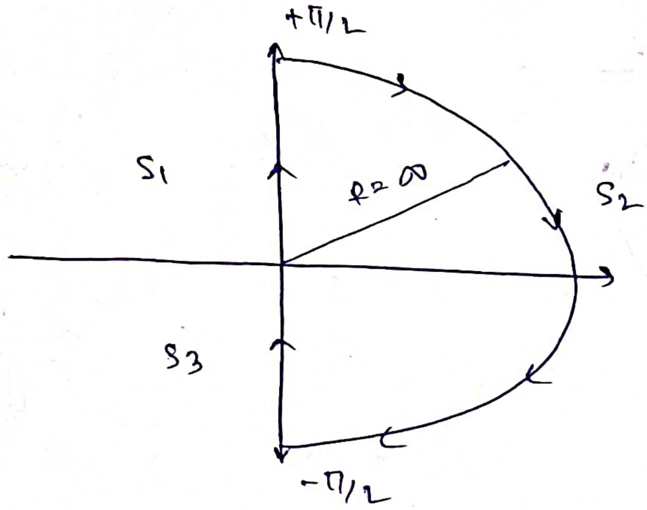
#→ → To Map S_4.

Put $s = \lim_{R \to 0} Re^{j\theta}$ (where $\theta = -\pi/2 \to \pi/2$)

#→ The Encirclement of the Nyquist Contour is ~~abg~~ about $(-1+0j)$ when apply to open loop transfer funcn.

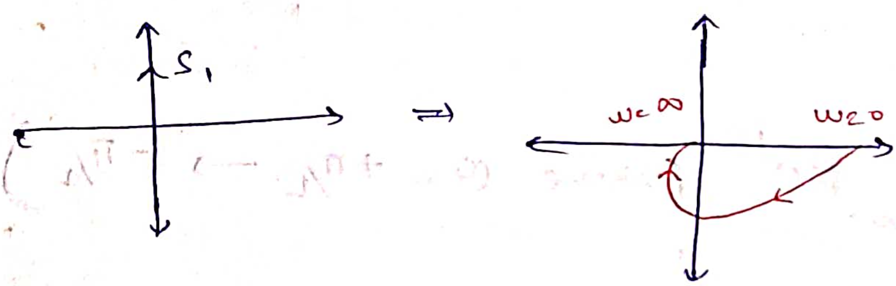
14/11/2008

Example 1 $G(s)H(s) = \dfrac{10}{(s+1)(s+4)}$



→ To map S_1

i.e polar plot



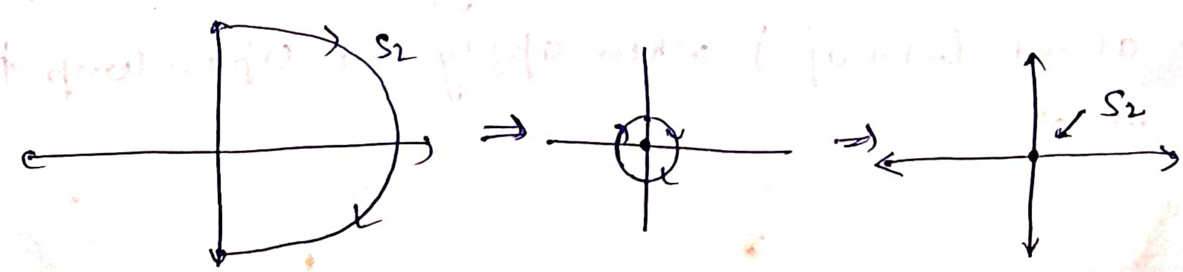
→ To map S_2

Put $s = \lim_{R \to \infty} Re^{j\theta}$ $[\theta = +\pi/2 \to -\pi/2]$

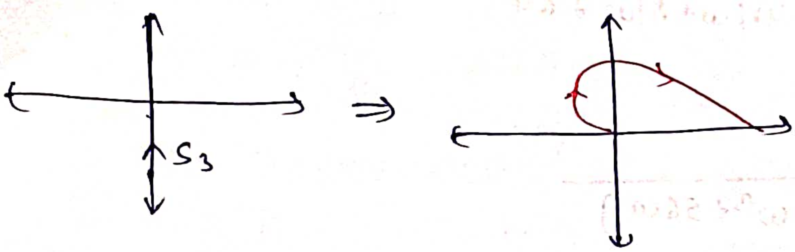
$G(s) = \dfrac{10}{s^2} = \dfrac{10}{\lim_{R\to\infty} R^2 e^{j2\theta}}$ (as 1, 4 << ∞ insignificant in compare to ∞)

$= 0e^{-j2\theta}$

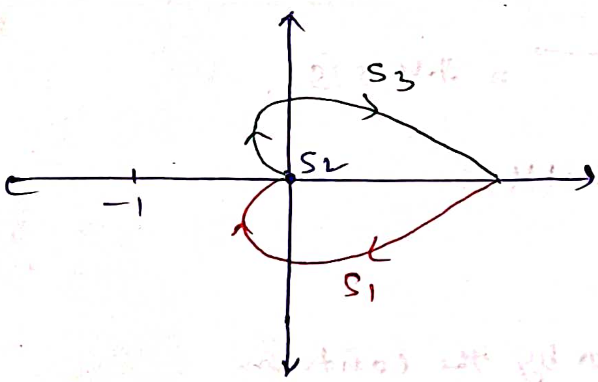
$\Rightarrow 0e^{-j\pi} \to 0e^{+j\pi}$

⇒ 

→ To map S_3 (Inv. polar plot)



∴ ⇒ Nyquist plot 1.



∴ ⇒ $N = 0$

⇒ $P = 0$

⇒ $N = P - Z$

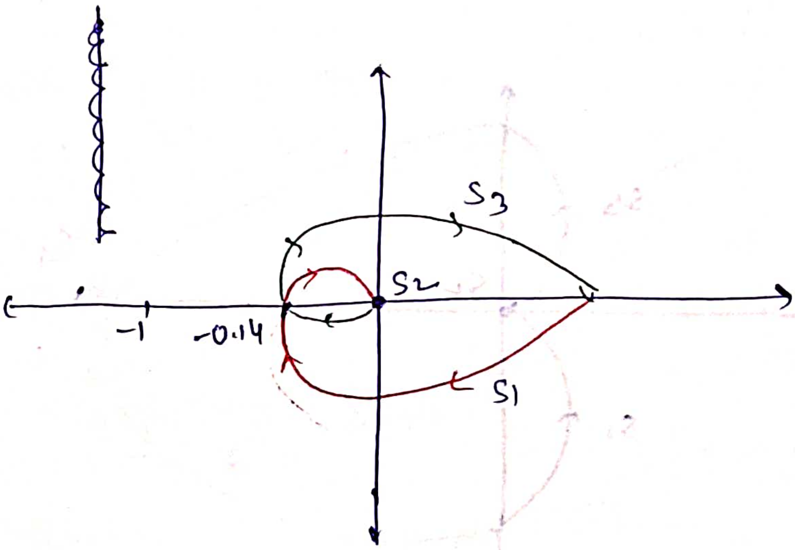
$0 = 0 - Z$

⇒ $\boxed{Z = 0}$

⇒ No, closed loop poles on R.H.S, and so, system is Adsolutely stable.

Ques: $G(s)H(s) = \dfrac{100}{(s+2)(s+4)(s+8)}$

Sol:-



Put $s = j\omega$ (to check the encirclement of "-1")

∴ $\dfrac{100}{(j\omega+2)(j\omega+4)(j\omega+8)} \Rightarrow \dfrac{100}{(-\omega^2+6j\omega+8)(j\omega+8)}$

$$\Rightarrow \frac{100}{-j\omega^3 - 8\omega^2 - 6\omega^2 + 48j\omega + 8j\omega + 64}$$

$$\Rightarrow \frac{100}{(64 - 14\omega^2) + j[-\omega^3 + 56\omega]}$$

$$-\omega^3 + 56\omega = 0$$

$$\Rightarrow \omega^2 = 56$$

$$\therefore \omega = \omega_{pc} = \sqrt{56} = 7.4\ r/s.$$

$$\Rightarrow \frac{100}{64 - 14(7.4)^2} = -0.14.$$

$\Rightarrow$ "-1" is not encircled ~~to~~ by the contour.

$\therefore \Rightarrow$ ~~Stab.~~ System is "Absolutely Stable".

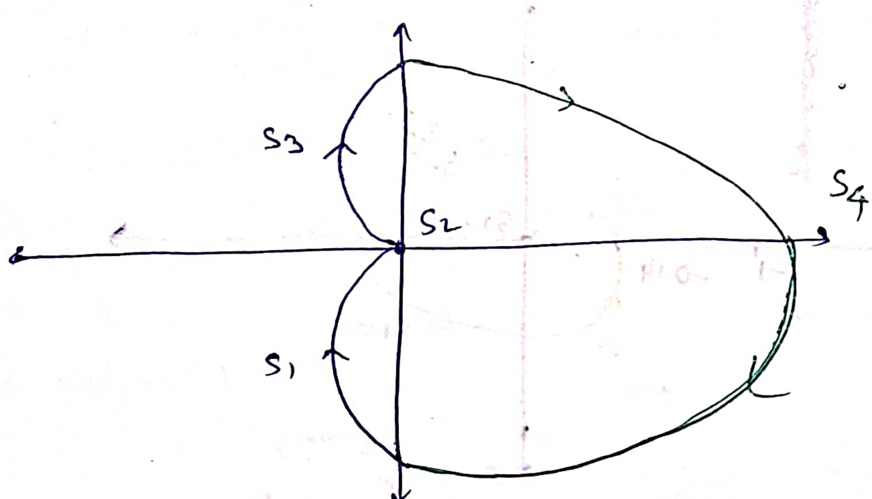
as $N = P - Z$

$0 = 0 - Z \Rightarrow \boxed{Z = 0}$

Quest

$$G(s) = \frac{1}{s(s+5)}$$

Soln



Now for Mapping of S_4

put $s = 0$

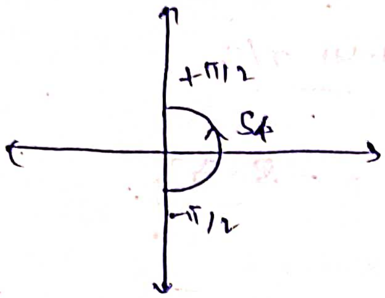
$$\Rightarrow G(s) = \frac{1/5}{s}$$ $\left[\begin{array}{l}\text{as } (s+5) \to 5 \\ \text{when } s \to 0\end{array}\right]$

$$= \frac{1/s}{\lim_{R\to 0} Re^{j\theta}} = \infty e^{-j\theta}$$

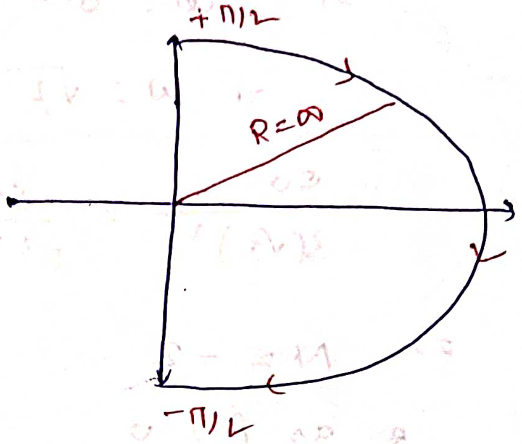
$\therefore \quad \theta \to -\pi/2 \to +\pi/2$

$\Rightarrow \quad G(s) \to \infty e^{j\pi/2} \to \infty e^{j-\pi/2}$

$\Rightarrow$



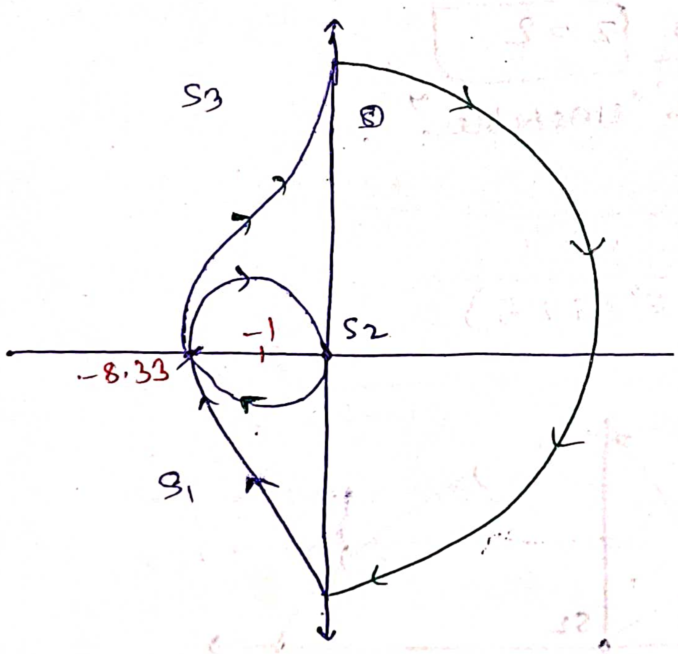
$\Rightarrow$



Ques.

$$G(s) = \frac{50}{s(s+1)(s+2)}$$

Sol^n.



$\rightarrow$ To Map S_4

$$G(s) = \frac{25}{s} = \frac{25}{\lim_{R\to 0} Re^{j\theta}} = \infty e^{-j\theta}$$

$[\theta \to -\pi/2 \to +\pi/2]$

$\therefore \; G(s) \Rightarrow \infty e^{j\pi/2} \to \infty e^{j-\pi/2}$.

$$\frac{50}{j\omega(j\omega+1)(j\omega+2)} = \frac{50}{(-\omega^2+j\omega)(j\omega+2)}$$

$$= \frac{50}{-j\omega^3 - 2\omega^2 - \omega^2 + 2\omega j} = \frac{50}{-3\omega^2 + j(2\omega - \omega^3)}$$

$$\Rightarrow 2\omega - \omega^3 = 0$$

$$\Rightarrow 2\omega = \omega^3$$

$$\Rightarrow \omega = \sqrt{2} = 1.41 \ r/s.$$

$$\frac{50}{-3(\sqrt{2})^2} = \frac{50^{25}}{-3 \times 2} = -8.33.$$

$$\Rightarrow N = -2$$

as $P = 0$

$$\Rightarrow -2 = P - Z$$

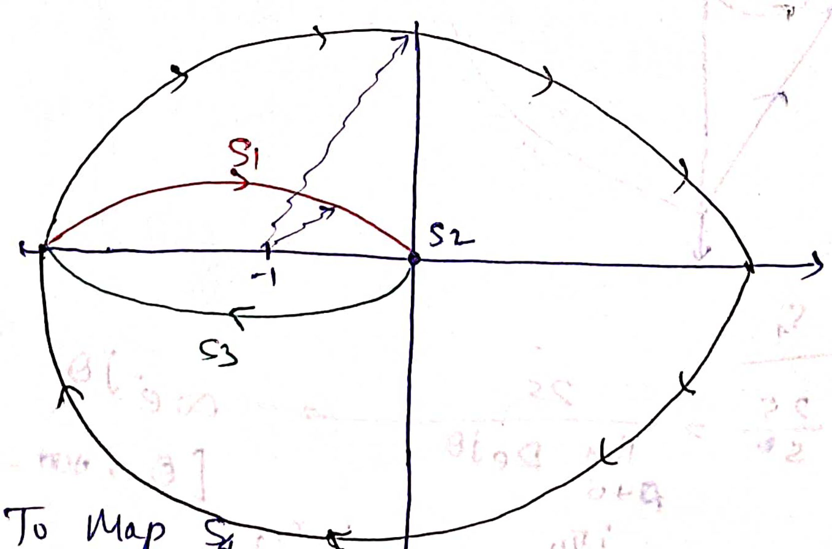
$$\Rightarrow \boxed{Z = 2}$$

So, the system is "unstable".

Quest.

$$G(s) = \frac{10}{s^2(s+5)}$$

Soln.



→ To Map S_1

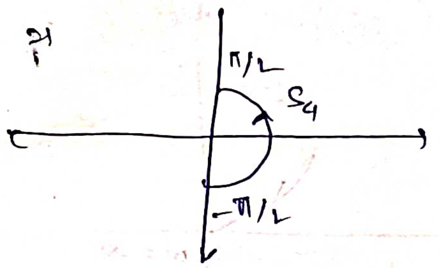
$$G(s) = \frac{10}{s^2[5]} = \frac{2}{s^2} = \frac{2}{\lim_{R \to 0} R^2 e^{j2\theta}}$$

$$= \infty e^{-j2\theta}.$$

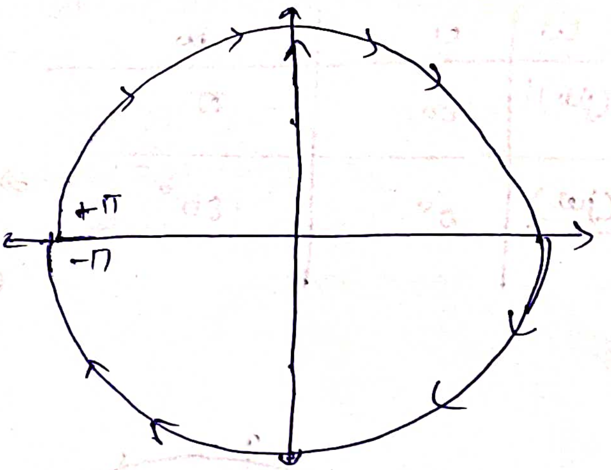
$\Rightarrow$ as $\theta \Rightarrow [-\pi/2 \rightarrow \pi/2]$

$\therefore \Rightarrow G(s) \Rightarrow [\infty e^{j\pi} \rightarrow \infty e^{-j\pi}]$

$\Rightarrow$



$\Rightarrow$



$\therefore \Rightarrow \quad N = -2$

$\Rightarrow P - Z = -2 \Rightarrow 0 - Z = -2$

$\Rightarrow \boxed{Z = 2}$

So, the system is "Unstable."

Ques:

$$G(s) = \frac{s+2}{s^2(s-1)}$$

$$G(s) = \frac{s+2}{s^2(s-1)}$$

$$= \frac{2[1+0.5s]}{-1s^2(1-s)}$$

$$= \frac{-2(1+0.5s)}{s^2(1-s)}$$

Put $s = j\omega$

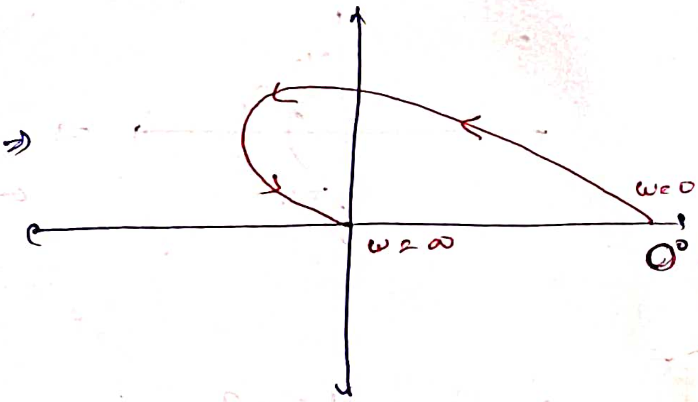
$$\Rightarrow G(j\omega) = \frac{-2(1+j0.5\omega)}{(j\omega)^2[1-j\omega]}$$

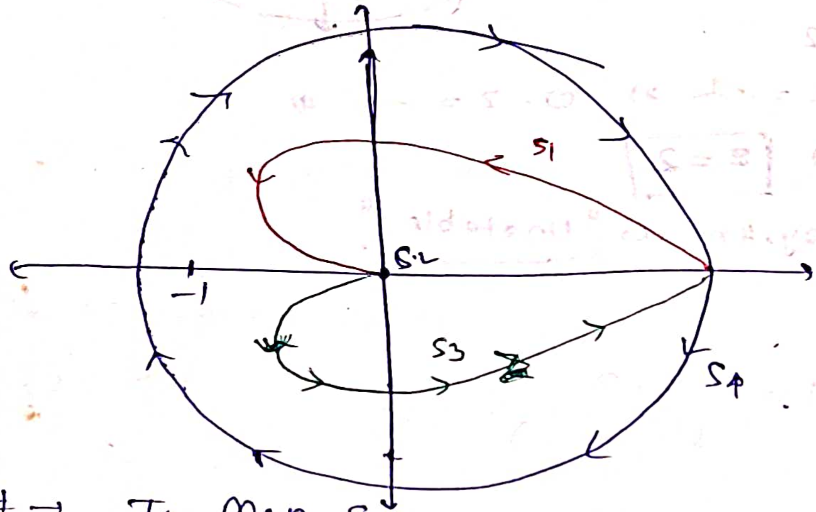
$$\Rightarrow |G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\omega^2\sqrt{1+\omega^2}}$$

and $\angle G(j\omega) = \frac{[180^\circ][\tan^{-1} 0.5\omega]}{[180^\circ][-\tan^{-1}\omega]} = \tan^{-1} 0.5\omega + \tan^{-1}\omega.$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	0°	180°

$\Rightarrow$





→ To Map S_4

$$G(s) = \frac{2}{s^2(-1)}$$

$$= \frac{2}{\lim_{R \to 0} R^2 e^{j2\theta} \cdot e^{j\pi}}$$

where $\boxed{-1 = e^{j\pi}}$,

$$= \infty e^{-j[2\theta + \pi]}$$

as $\theta = -\pi/2 \rightarrow +\pi/2$

$$G(s) = \infty e^{-j[0]} \longrightarrow \infty e^{-j[2\pi]}$$

$\Rightarrow$ Angle $\rightarrow$ $\boxed{0 \longrightarrow -360^\circ}$

$-1 = 1 - Z$

$\boxed{Z = 2}$ ⇒ System is "Unstable".

Ques 2

$$G(s) = \frac{K(1+s)^2}{s^3}$$

find the range of "K" for stability.

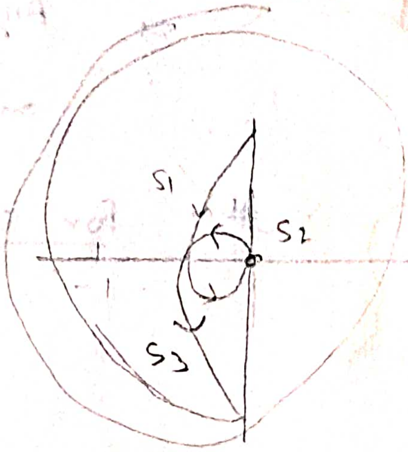
Sol^n.

$$G(j\omega) = \frac{K(1+j\omega)^2}{(j\omega)^3}$$

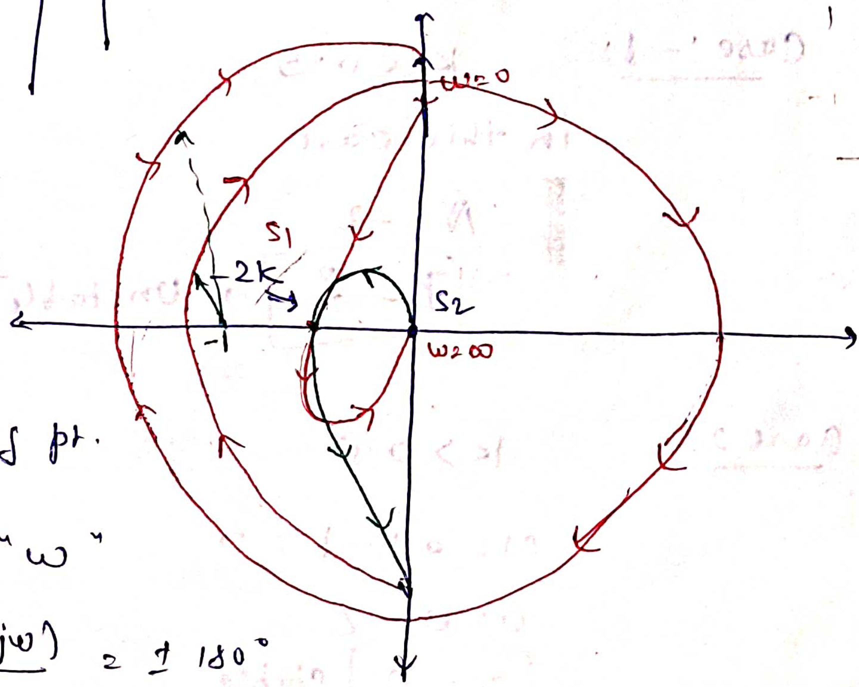
$$|G(j\omega)| = \frac{K(1+\omega^2)}{\omega^3}$$

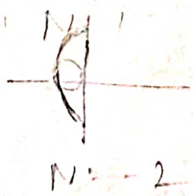
$$\angle G(j\omega) = \frac{0\,[2\tan^{-1}\omega]}{270^\circ}$$

$$\angle G(j\omega) = -270^\circ + 2\tan^{-1}\omega$$



ω	0	1	∞
$\lvert G(j\omega)\rvert$	∞	2K	0
$\angle G(j\omega)$	-270°	-180°	-90°





#→ To find the cutting pt. on ~~im~~ real axis

put such value of "ω"

such that $\angle G(j\omega) = \pm 180^\circ$

so, Here on putting $\omega = 1$

$\boxed{\angle G(j\omega) = -180^\circ}$

And putting $\omega = 1$

$$\boxed{|G(j\omega)| = 2K.} \quad \text{— (1)}$$

→ To map for S_4

$s \to 0$

$$\Rightarrow G(s) = \frac{K}{s^3} = \frac{K}{\lim_{R \to 0} R^3 e^{j3\theta}}$$

$$G(s) = \infty e^{-j3\theta} \quad [\theta \Rightarrow -\pi/2 \to \pi/2]$$

$\Rightarrow$ Angle vary as → $-\frac{3\pi}{2} \to +\frac{3\pi}{2}$

$\Rightarrow$ $G(s)$ vary as $\Rightarrow$ $\infty e^{+j\frac{3\pi}{2}} \to \infty e^{-j\frac{3\pi}{2}}$

#→ for system to be ~~magn~~ marginally stable

$-2K = -1$

$\Rightarrow$ ~~K~~ $\boxed{K_{mar} = 0.5}$

Case :- 1 :- $K < 0.5$

in this case

$N = -2$

$\Rightarrow \boxed{Z = 2}$ → Unstable

Case 2 :- $K > 0.5$

$N = 1 - 1 = 0$

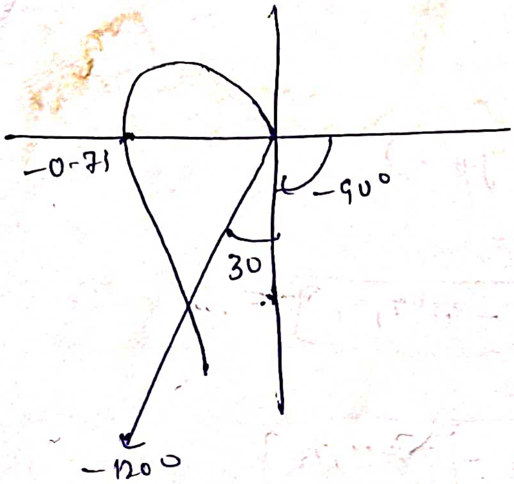
$0 = 0 - Z$

$\boxed{Z = 0}$ stable

$\Rightarrow \boxed{K > 0.5}$ Ans

W.B Ex: -6

Quest (3) :-



$\therefore$ $\boxed{P.M = 180° - 120 = 60 \qquad G.M = \frac{1}{0.75}.}$

(7) :-

(a) $$\frac{K}{(1+j\omega)(1+2j\omega)(1+3j\omega)}$$

$$= \frac{K}{(1+2j\omega+j\omega-2\omega^2)(1+3j\omega)} = \frac{K}{(1+3j\omega-2\omega^2)(1+3j\omega)}$$

$$= \frac{K}{1+3j\omega-2\omega^2+3j\omega - 9\omega^2 - 6j\omega^3}$$

$$= \frac{K}{(1-2\omega^2-9\omega^2)+j(3\omega+3\omega-6\omega^3)}$$

$$= \frac{K}{(1-11\omega^2)+j(6\omega-6\omega^3)}$$

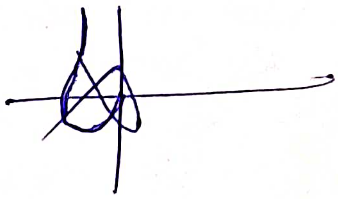
$$6\omega - 6\omega^3 = 0$$

$$\cancel{6\omega} = \cancel{6}\omega^{\cancel{3}\,2}$$

$$\omega^2 = 1$$

$$\omega = 1$$

$\therefore$ $\omega_{pc} = 1.$



9) :-

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

$$= \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \frac{1}{j\omega[1+j\omega T_2+j\omega T_1-\omega^2 T_1 T_2]}$$

$$= \frac{1}{[j\omega-\omega^2 T_2-\omega^2 T_1-j\omega^3 T_1 T_2]}$$

$$= \frac{1}{-\omega^2(T_1+T_2)+j(\omega-\omega^3 T_1 T_2)}$$

$$\omega-\omega^3 T_1 T_2 = 0$$

$$\omega^2 = \frac{1}{T_1 T_2}$$

$$\omega = \frac{1}{\sqrt{T_1 T_2}} = \omega_{pc}$$

$$\frac{-1}{\omega^2(T_1+T_2)} = \frac{-1}{\frac{1}{T_1 T_2}(T_1+T_2)}$$

$$\boxed{x = \frac{T_1 T_2}{T_1+T_2}}$$

$$\therefore \boxed{G.M = \frac{1}{x} = \frac{T_1+T_2}{T_1 T_2}.}$$

17) → $$P.M = \tan^{-1}\frac{2\zeta}{\left(-2\zeta^2+\sqrt{1+4\zeta^4}\right)^{1/2}}$$

$$\approx 100\,\zeta$$