Introduction of State Space Analysis of Control system :-

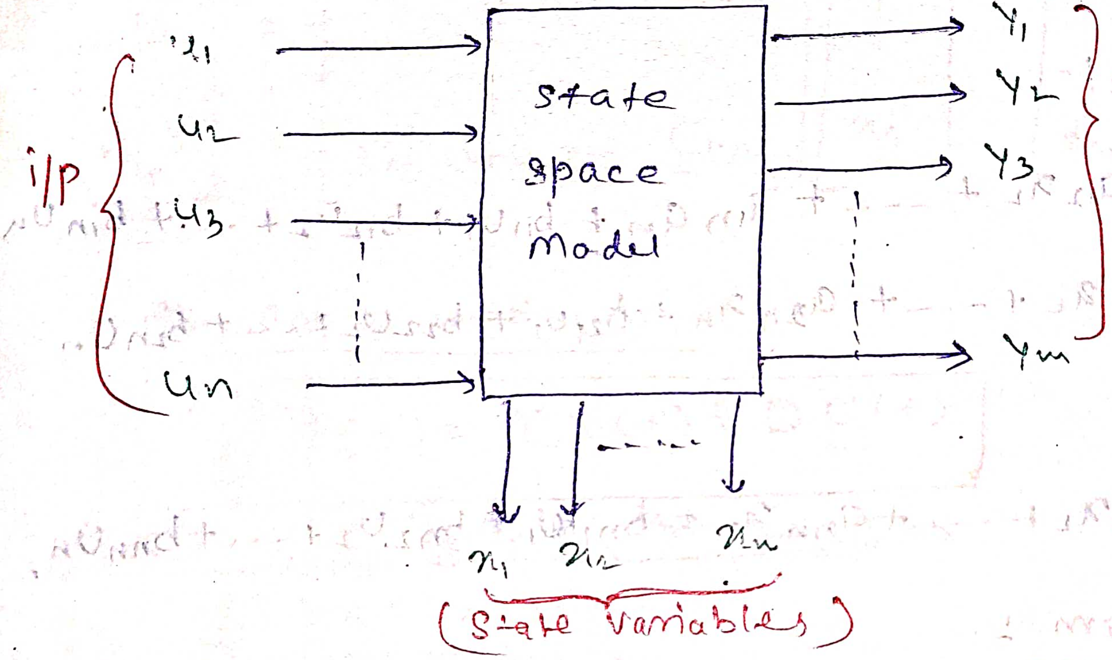
The analysis of control system carried out so far is based on transfer functions and graphical approach such as root locus, Bode and Nyquist plot. The i/p o/p relations are in the form of transfer function. In this approach of analysis initial conditions are considered zero meaning that the system is initially at rest and the time solution obtained is in a general form due to i/p only. However, for multiple i/ps multiple o/ps and systems initially not at rest, the T.F approach for ~~any~~ analysis is inadequate and not convenient.

The use of state space approach for the any analysis of control systems enables to overcome the shortcomings of T.F approach. The state space approach to be described hereunder can be conveniently used for analysis of control system with basic knowledge of matrix algebra.

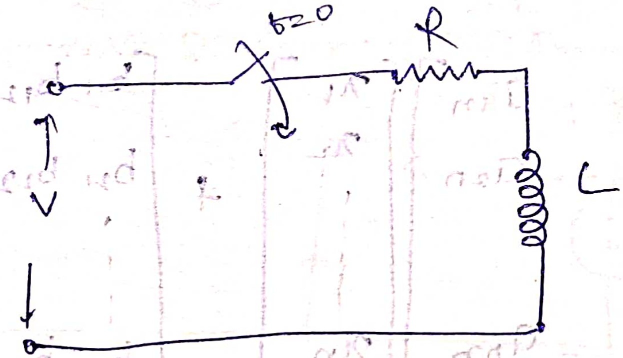
The analysis of a control system using state space approach carried out in time domain by representing a system in the form of first order differential equations by selecting suitable state variables where in first order derivative terms are arranged on left hand side and right hand side terms are free from derivations. The variables in such a form of representation are known as State variables. In state space approch multiple - i/p - multiple o/p system equation can be arranged in matrix form which facilitates their solution.

{18/11/2008}

STATE SPACE ANALYSIS :-



Example :



At $t=0^-$

$i_L(0)^- = 0$ Amps

$i(0) = 0$ Amps.

At $t = 0^+$

$$i_L(0)^+ = \frac{1}{L}\int_{t=0}^{t=0^+} V dt = 0 \text{ Amps}.$$

$$V = iR + L\frac{di}{dt}.$$

$$\frac{di}{dt} = \frac{V}{L} - i\frac{R}{L}\left[A/s\right] \quad \text{—①}$$

→ equ. ① represents, state equ of electrical n/w

→ ~~Space~~ State Space model consist of two equn; ① the first one "state equation". ① Ist derivative of state variables expressed as a linear combination of state variables and i/ps.

State Equation :-

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}a_n + b_{11}u_1 + b_{12}u_2 + \dots + b_{1n}u_n$$

$$x_2^{\circ} = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + b_{21}U_1 + b_{22}U_2 + \dots + b_{2n}U_n$$

$$\vdots$$

$$x_n^{\circ} = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_{n1}w_1 + b_{n2}U_2 + \dots + b_{nn}U_n,$$

In matrix form :-

$$\begin{bmatrix} x_1^{\circ} \\ x_2^{\circ} \\ \vdots \\ x_n^{\circ} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & & & \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

$$\therefore \boxed{X^{\circ}(t) = AX(t) + BU(t)}$$

Output Equation :-

#→ Outputs at any instants of time are expresed as linear combination of state variables and i/ps. And we write the output equation as:-

$$y_1 = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n + d_{11}U_1 + d_{12}U_2 + \cdots + d_{1n}U_n$$

$$y_2 = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n + d_{21}U_1 + d_{22}U_2 + \cdots + d_{2n}U_n$$

$$\vdots$$

$$y_n = c_{n1}x_1 + c_{n2}x_2 + \cdots + c_{nn}x_n + d_{n1}U_1 + d_{n2}U_2 + \cdots + d_{nn}U_n .$$

In Matrix Form :-

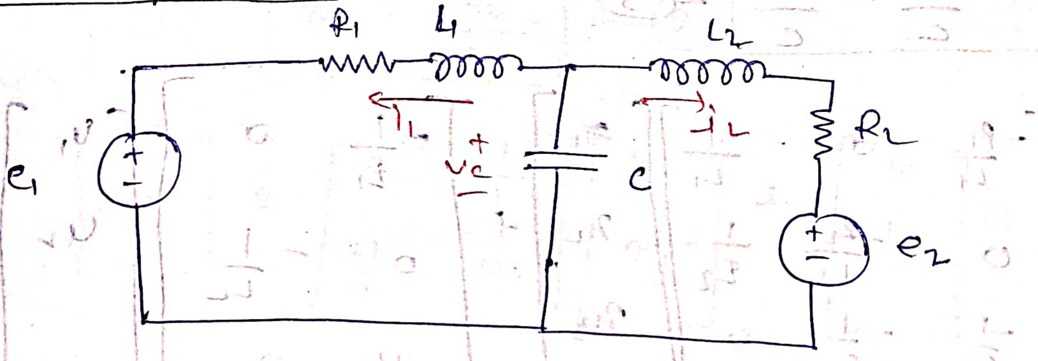
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & & \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & & & \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

this can be written as

$$\boxed{Y(t) = CX(t) + DU(t)}$$

Different Types of Problems :- ⇒

Type 1:- problems :-



① :- Choose state variables :-

$x_1 = i_1$; $x_2 = i_2$; $x_3 = V_c$

② :- Inputs :- $u_1 = e_1$; $u_2 = e_2$

output $y = V_c$.

③:- **Analysis** :-

Apply KVL to loop ①

$$L_1 \frac{di}{dt} + i_1 R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt} = -i_1 \frac{R_1}{L_1} + \frac{V_c}{L_1} - \frac{e_1}{L_1}$$

$$\boxed{\dot{x}_1 = -x_1 \cdot \frac{R_1}{L_1} + \frac{x_3}{L_1} - \frac{u_1}{L_1}} \quad \text{—①}$$

Loop ② :-

$$L_2 \frac{di_2}{dt} + R_2 I_2 + e_2 - V_c = 0$$

$$\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 + \frac{V_c}{L_2} - \frac{e_2}{L_2}$$

$$\boxed{\dot{x}_2 = -\frac{R_2}{L_2} x_2 + \frac{x_3}{L_2} - \frac{u_2}{L_2}} \quad \text{—②}$$

KCL at Node V_c :-

$$i_1 + i_2 + C\frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} = -\frac{i_1}{C} - \frac{i_2}{C}$$

$$\boxed{\dot{x}_3 = -\frac{x_1}{C} - \frac{x_2}{C}} \quad \text{—③}$$

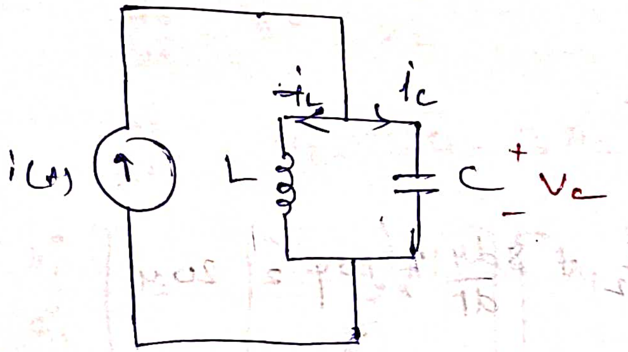
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

O/P equ

$$y = V_c = x_3$$

i.e ⇒ $y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Quest→



~~let $x_1 = i_L$~~
~~$x_2 = V_c$~~
~~$u = i(t)$~~

Sol:- → $i(t) = i_L + i_c$

let:-
$x_1 = i_L$
$x_2 = V_c$
$u = i(t)$

$$i(t) = i_L + C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{i(t)}{C} - \frac{1}{C} \cdot i_L$$

$$\boxed{\overset{\circ}{x}_2 = -\frac{1}{C}x_1 + \frac{u}{C}} \quad — ①$$

→ Apply KVL to the loop :-

$$L\frac{di_L}{dt} - V_c = 0$$

$$\cancel{dt}\ \frac{di_L}{dt} = \frac{V_c}{L}$$

$$\boxed{\overset{\circ}{x}_1 = \frac{x_2}{L}} \quad — ②$$

∴ ⇒
$$\begin{bmatrix} \overset{\circ}{x}_1 \\ \overset{\circ}{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/c \end{bmatrix} u$$

let $y = i_L = x_1$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Type 2 problems:⇒ To obtain State models from differential equn:

① ~~Choose State~~ variables

let:- ~~$y = x_1$~~

$\therefore \frac{dy}{dt} = \dot{x}_1 = x_2$

d2

$$\frac{d^3y}{dt} + 4\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 10y = 20u$$

Choose State variables

let :- $y = x_1$

$\therefore \frac{dy}{dt} = \dot{x}_1 = x_2$

$\frac{d^2y}{dt^2} = \dot{x}_2 = x_3$

$\frac{d^3y}{dt^3} = \dot{x}_3$

$\therefore \dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = 20u - 4x_3 - 8x_2 - 10x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

O/p equ:

let $y = x_1$

$\Rightarrow y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

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②

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u(t)$$

let $y = x_1$

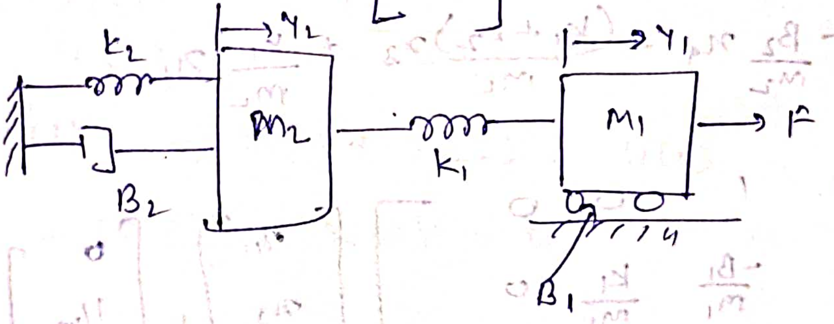
$$\frac{dy}{dt} = \dot{x}_1 = x_2$$

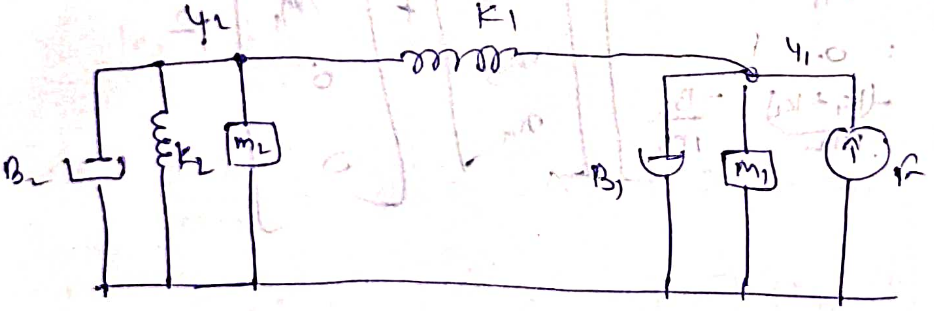
$$\frac{d^2y}{dt^2} = \dot{x}_2 = u - 3x_2 - 2x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

o/p:- $y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

③





At node y_1:- $f = M_1\frac{d^2y_1}{dt^2} + B_1\frac{dy}{dt} + K_1(y_1 - y_2)$

$$\frac{F}{M_1} = \frac{d^2y_1}{dt^2} + \frac{B_1}{M_1}\frac{dy_1}{dt} + \frac{K_1}{M_1}\cdot y_1 - \frac{K_1}{M_1}y_2 \quad ①$$

At node y_2:-

$$0 = M_2\frac{d^2y_2}{dt^2} + B_2\frac{dy_2}{dt} + y_2K_2 + K_1y_2 - K_1y_1$$

$$0 = \frac{d^2y_2}{dt^2} + \frac{B_2}{M_2}\frac{dy_2}{dt} + \frac{y_2[K_1+K_2]}{M_2} - \frac{K_1}{M_2}y_1 \quad ②$$

Choose state variables :

let $y_1 = x_1$ let $y_2 = x_3$

$\Rightarrow \frac{dy_1}{dt} = \dot{x}_1 = x_2$ $\frac{dy_2}{dt} = \dot{x}_3 = x_4$

$\frac{d^2 y_1}{dt^2} = \dot{x}_2$ $\frac{d^2 y_2}{dt^2} = \dot{x}_4$

$\therefore \dot{x}_1 = x_2$

$\dot{x}_2 = \frac{F}{M_1} - \frac{B_1}{M_1} x_2 - \frac{K_1}{M_1} x_1 + \frac{K_1}{M_1} x_3$

$\dot{x}_3 = x_4$.

and $\dot{x}_4 = -\frac{B_2}{M_2} x_4 - \frac{(K_1+K_2)}{M_2} x_3 + \frac{K_1}{M_2} x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{M_1} & -\frac{B_1}{M_1} & \frac{K_1}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_1}{M_2} & 0 & -\frac{(K_1+K_2)}{M_2} & -\frac{B_2}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix} F$$

o/p equn

$y_1 = x_1$

$y_2 = x_3$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

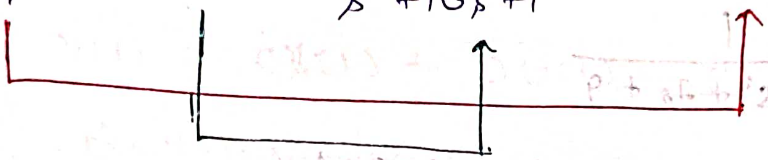
Type 3 problems :- Phase variable form

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + y = 5\frac{du}{dt} + 10u,$$

$$s^2Y(s) + 10sY(s) + Y(s) = 5s\,U(s) + 10U(s).$$

$$\frac{Y(s)}{U(s)} = \frac{5s+10}{s^2+10s+1}$$

$$\frac{Y(s)}{X(s)} \cdot \frac{X_1(s)}{U(s)} = \frac{1}{s^2+10s+1}\left[5s+10\right]$$



Now:- $$\frac{X_1(s)}{U(s)} = \frac{1}{s^2+10s+1}$$

$$\Rightarrow s^2X_1(s) + 10s\,X_1(s) + X_1(s) = U(s)$$

$$\Rightarrow \frac{d^2x_1}{dt^2} + 10\frac{dx_1}{dt} + x_1(t) = u$$

let $\frac{dx_1}{dt} = x_2$

$$\therefore \frac{d^2x_1}{dt^2} = \dot{x}_2 = u - 10x_2 - x_1$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$$

O/P side:-

$$\frac{Y(s)}{X_1(s)} = 5s+10$$

$$Y(s) = 5s\,X_1(s) + 10X_1(s)$$

$$y = 5\frac{dx_1}{dt} + 10x_1$$

$$y = 10x_1 + 5x_2$$

$$y = \begin{bmatrix} 10 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

W.B Ex 18

Q15

$$G(s) = \frac{2s+1}{s^2+7s+9}$$

$$\frac{Y(s)}{U(s)} = \frac{Y(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = \frac{1}{s^2+7s+9} \cdot [2s+1]$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^2+7s+9}$$

$$s^2 X_1(s) + 7s X_1(s) + 9 X_1(s) = U(s)$$

$$\frac{d^2x_1}{dt^2} + 7\frac{dx_1}{dt} + 9x_1 = u$$

let $\frac{dx_1}{dt} = x_2$

$$\Rightarrow \frac{d^2x_1}{dt^2} = \dot{x}_2 = u - 7x_2 - 9x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\frac{Y(s)}{X_1(s)} = (2s+1)$$

$$y = 2s X_1(s) + X_1(s)$$

$$y = 2\frac{dx_1}{dt} + x_1$$

$$y = 2x_2 + x_1$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Type 4:- problems:- To obtain transfer function from state models.

State models

$$x^{\circ}(t) = \cancel{A\text{-}Ex}\ A x(t) + B u(t)$$

$$y(t) = C x(t) + D u(t)$$

Applying Laplace Transform

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s) \quad — ①$$

For transferfuncn. :- $X(0) = 0$

$$sX(s) - AX(s) = BU(s)$$

$$\Rightarrow X(s)\left[sI - A\right] = BU(s)$$

$$\Rightarrow X(s) = (sI - A)^{-1} B U(s) \quad — ②$$

put ② in ①

$$\therefore Y(s) = C\left[sI - A\right]^{-1} B U(s) + DU(s).$$

$$\therefore Y(s) = \left\{ C\left[sI - A\right]^{-1} B + D \right\} U(s)$$

$$\Rightarrow \boxed{\frac{Y(s)}{U(s)} = \underbrace{C\left[sI - A\right]^{-1} B + D}_{\text{Transfer matrix.}}}$$

Type 5 - problems

~~Stab~~ Stability of s.m.'s

$$\frac{Y(s)}{U(s)} = \frac{C \, \mathrm{Adj}(sI - A)}{|sI - A|} \cdot B + D$$

$$\frac{Y(s)}{U(s)} = \frac{C\,Adj\,(sI - A)\cdot B + |sI - A|\,D}{|sI - A|.}$$

The roots of

$$\left.\begin{array}{l} 1+G(s)H(s) = 0 \\ \text{or C.L poles} \end{array}\right\} \Rightarrow |sI - A| = 0$$

$\boxed{\text{Eigen Values of Sys. matrix } [A] = \text{C.L poles.}}$

Type 6:- problems:-

Controllability and obserlabity.

#- Controllability implies the state variables choosen can be controlled or not by some controlled vector in finite time "t".

#- Observability implies ~~majarbility~~ measurability of ~~space~~ state variables

KALMAN'S TEST :-

& for Controllability

$$Q_c = [\,B \quad AB \quad A^2B \; \dots\dots \; A^{n-1}B\,]$$

For Observability

$$Q_o = [\,C^T \quad A^TC^T \quad (A^T)^2C^T \dots\dots (A^T)^{n-1}C^T\,]$$

#→ For state ~~vara~~ variables to be Controllable and observable :-

$$|Q_c| \neq 0 \quad \text{and} \quad |Q_o| \neq 0$$

Type :- 7 :- Problems :-

Solution of State equn.

#→ State Equn.

$$\dot{X}(t) = A X(t) + B U(t)$$

(i) :- Free Response [i.e. u(t) = 0]

$$\frac{dX(t)}{dt} - A X(t) = 0 \quad \text{—①}$$

solun. of above diff. equn. is

$$X(t) = k e^{At} \quad \text{—②}$$

Apply L.T to equ. ①

$$[SI - A] X(s) = X(0)$$

$$\Rightarrow X(s) = (SI - A)^{-1} X(0)$$

$$X(t) = \{L^{-1}(SI - A)^{-1}\} X(0)$$

Compared as − $X(t) = e^{At}\, k$

$$\therefore \boxed{\phi(t) = e^{At} = L^{-1}(SI - A)^{-1}}$$

→ Soln of free response.

state Transition Matrix

(ii) :- Forced Response :-

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$(SI - A)X(s) = X(0) + BU(s)$$

$$X(s) = (SI - A)^{-1} X(0) + (SI - A)^{-1} \cdot B \cdot U(s)$$

$$\boxed{\therefore X(t) = \left\{ L^{-1}(SI - A)^{-1} \right\} X(0) + L^{-1}\left[(SI - A)^{-1} \cdot B \cdot U(s) \right]}$$

solution of force response

W.B

12×L-18

① L

P] $S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$

$\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$

$SI - A = \begin{bmatrix} (S+1) & -2 \\ 0 & (S+2) \end{bmatrix}$

$|SI - A| = (S+1)(S+2) = 0$

$S = -1, -2,$

Q] $S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$

$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$

$SI - A = \begin{bmatrix} (S+1) & 2 \\ 2 & S+4 \end{bmatrix}$

$|SI - A| = (S+1)(S+4) - 4 = 0$

$S^2 + 5S + 4 - 4 = 0$

$s(s+5) = 0$

$s = 0, -5.$

2] : $s\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} - \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$|sI - A| = \begin{bmatrix}s & 1\\ -1 & s\end{bmatrix} \Rightarrow s^2 + 1 = 0$

$s = \pm j1$

③ $\dot{x} = AX(t) + Bu(t)$

$\therefore A = \begin{bmatrix}0 & 1\\ 2 & -3\end{bmatrix} \quad B = \begin{bmatrix}0\\ 1\end{bmatrix}$

$Q_c = \begin{bmatrix}B & AB\end{bmatrix}$

$AB = \begin{bmatrix}0 & 1\\ 2 & -3\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -3\end{bmatrix}$

$Q_c = \begin{bmatrix}0 & 1\\ 1 & -3\end{bmatrix} = 0$

$\therefore |Q_c| = -1$ controllable

$Q_o = \begin{bmatrix}C^T & A^T C^T\end{bmatrix}$

$A^T C^T = \begin{bmatrix}0 & 2\\ 1 & -3\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}2\\ -2\end{bmatrix}$

$Q_o = \begin{bmatrix}1 & 2\\ 1 & -2\end{bmatrix} = -4 = |Q_o|$

obseravable

④ →

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(~~SI-A~~) $u = \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix} x + v$

$$x^{\circ} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -0.5 & -3 & -5 \end{bmatrix} \right\} x + v$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.5 & -3 & -5 \end{bmatrix} x + v$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + v$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\therefore SI - A = S \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & (s+3) \end{bmatrix}$$

$$\therefore |sI - A| = s\{s(s+3) + 2\} + 1\{0\} + 0 .$$

$$= s^2(s+3) + 2s$$

$$= s^3 + 3s^2 + 2s$$

$$= \cancel{s^2(s}$$

$$= s^3 + s^2 + 2s^2 + 1$$

$$= s^2(s+1) + 2$$

$$= s(s^2 + 3s + 2)$$

$$= s^2 + 3s + 2 = 0$$

$$s^2 + 2s + s + 2$$

$$s(s+2) + 1$$

$|s = -1$

$-1 + 3 + 2$

$(s+1) = 0$

⑤ 1.

$$\dot{x}(t) = -2x(t) + 2u(t)$$

$$y(t) = 0.5x(t)$$

$$sX(s) = -2X(s) + 2U(s)$$

$$X(s)\{s+2\} = 2U(s)$$

$$X(s) = \frac{2}{s+2} U(s)$$

$$Y(s) = 0.5X(s)$$

$$Y(s) = 0.5 \times \frac{2}{(s+2)} \times U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s+2}$$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\text{Adj}[SI - A] = \begin{bmatrix} (s-1) & 1 \\ 0 & (s-1) \end{bmatrix} \Rightarrow \begin{bmatrix} (s-1) & 0 \\ 1 & (s-1) \end{bmatrix}$$

$$|SI - A| = (s-1)^2$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

$$\therefore \phi(t) = L^{-1}(SI - A)^{-1} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times1}$$

$$x(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

$$\boxed{\begin{array}{c} \phi(t) = e^{At} \\ At\ t = 0 \\ e^0 = I \end{array}}$$

→ Check for correctness of $\phi(t)$

⑪ :-

$$\frac{dw}{dt} = -w + i_q$$

$$\frac{di_q}{dt} = -\omega - 10 i_q + 10 u$$

$$s\omega(s) = -\omega(s) + i_a(s)$$

$$i_a(s) = (s+1)\omega(s) \quad - ①$$

$$s i_a(s) = -\omega(s) - 10 i_a(s) + 10U(s)$$

$$s(s+1)\omega(s) = -\omega(s) - 10(s+1)\omega(s) + 10U(s)$$

$$\{s^2+s + 10s + 10 + 1\}\omega(s) = 10U(s)$$

$$(s^2 + 11s + 11)\omega(s) = 10U(s)$$

$$\boxed{\frac{\omega(s)}{U(s)} = \frac{10}{s^2 + 11s + 11}}$$

⑬ :-

$$|sI - A| \rightarrow 1 + G(s)H(s)$$

⑯ :-

$$Q_0 = [C^T \quad A^T C^T]$$

$$A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} b & b \\ 0 & 2b \end{bmatrix}$$

$$|Q_0| = 2b^2$$

$$\therefore |Q_0| \neq 0$$

for all non-zero values of "b".

⑰:- $C[SI - A]^{-1}.B + D$

$$(SI - A)^{+1} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} (s+1) & 0 \\ 0 & (s+2) \end{bmatrix}$$

$$|SI - A| = (s+1)(s+2)$$

$$\text{Adj}\,[SI - A] = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix} = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$(SI - A)^{-1}.B = \begin{bmatrix} \frac{1}{(s+1)} & 0 \\ 0 & \frac{1}{(s+2)} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C[SI - A]^{-1}.B = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix} = \frac{1}{(s+1)}$$

⑫:- $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$ $\quad$ $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

$x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\quad$ $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$x(t) = e^{At}.x(0)$$

$$e^{-At}x(t) = e^{-At}.e^{At}x(0)$$

$$e^{-At}x(t) = x(0)$$

let $e^{-At} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow \begin{aligned} ae^{-2t} - 2be^{-2t} &= 1 \quad ① \\ ce^{-2t} - 2de^{-2t} &= -2 \quad ② \end{aligned}$$

From the second response

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \begin{aligned} ae^{-t} - be^{-t} &= 1 \quad ③ \\ ce^{-t} - de^{-t} &= -1 \quad ④ \end{aligned}$$

Solve to get $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$e^{-At} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e^{-a_1t} - e^{-b_1t} & e^{-a_2t} + e^{-b_2t} \\ -e^{-a_3t} - e^{-b_3t} & e^{-a_4t} - e^{-b_4t} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{a_1t} - e^{b_1t} & e^{a_2t} + e^{b_2t} \\ -e^{a_3t} - e^{b_3t} & e^{a_4t} - e^{b_4t} \end{bmatrix}$$