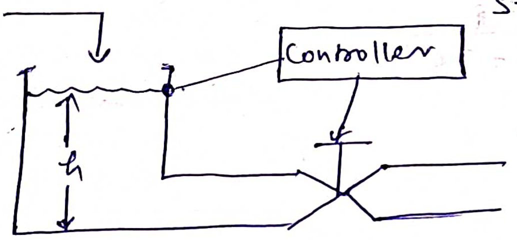
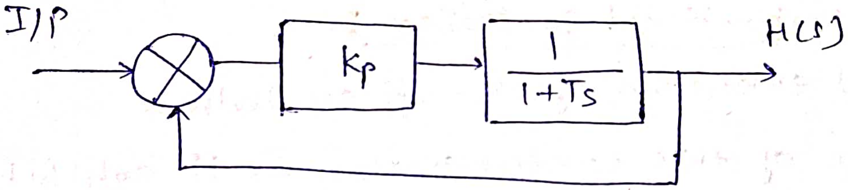
12/11/2008 :-

Industrial Controllers :→



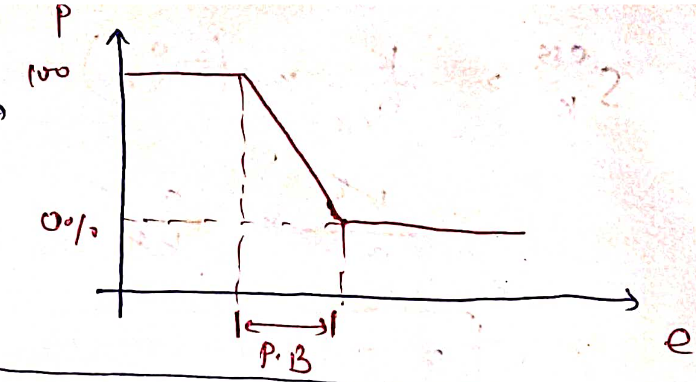


#→ Proportional Controller mode :-

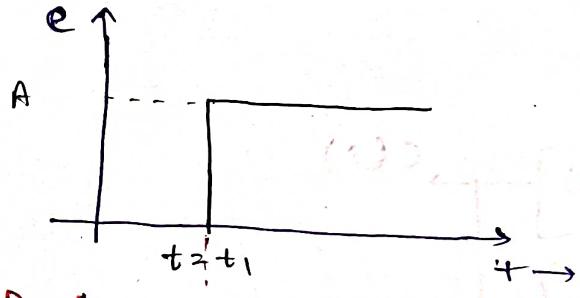
$$P \propto e$$

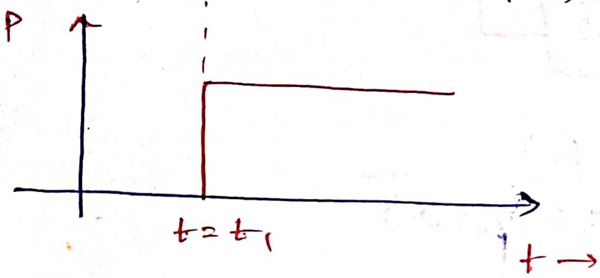
$\Rightarrow P = K_p e$ (where K_p = Proportional Gain)

so, $\boxed{P(s) = K_p E(s)}$



#→ $\boxed{\text{Proportional Band} = \frac{100}{K_p}}$





$$P = K_p A \quad [\text{as } e = A]$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{K_p}{(1+Ts)}}$$

$$e_{ss} = \frac{A}{1+K_p}$$

(offset)

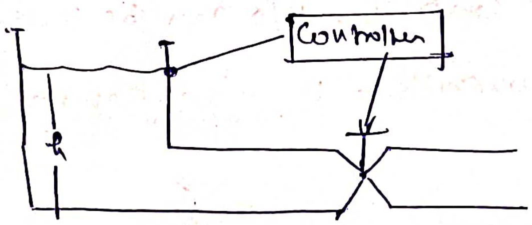
Imp. pts. about proportional controller :-

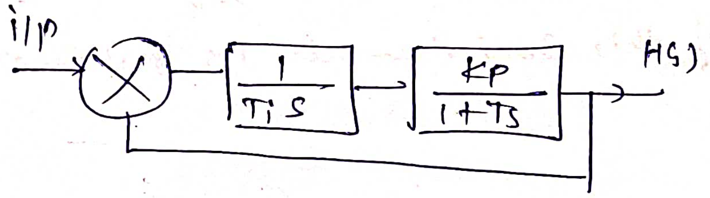
#→ It is natural extention of on-off controller

#→ Disadvantage of this control action is it exhibits permanent residual error known as offset at its operating point.

#→ A large gain kp implies less offset but also a narrow "Proportional Band", which may convert the proportional action into only on-off action..

Integral controller mode :





$$P \propto \int e\,dt$$

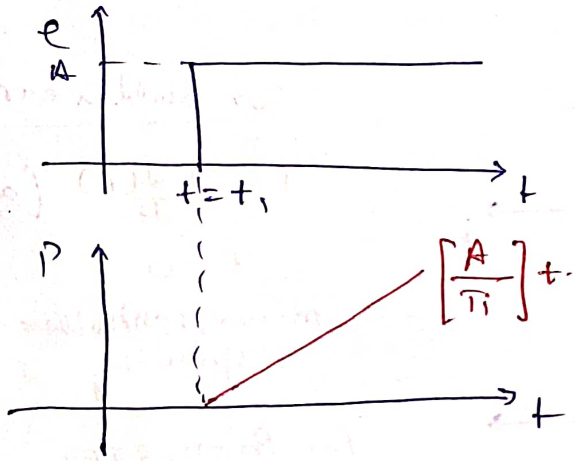
$$\Rightarrow P = K_I \int e\,dt \quad (\text{where } K_I \rightarrow \text{Intergoal scaling})$$

→ Defining Integral Time or Reset time :

$$\boxed{T_i = \frac{1}{K_I}}$$

$$P = \frac{1}{T_i}\int e\,dt.$$

$$\boxed{P(s) = \frac{1}{T_i\,S}\,E(s)} \Rightarrow E(s) \rightarrow \boxed{\frac{1}{T_i\,S}} \rightarrow P(s)$$



$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s}}{1 + \frac{1}{T_i\,S\,(1+T_s)}}$$

$$e_{ss} = \frac{A}{\infty} = 0$$

$$P = \frac{1}{T_i}\int A\,dt \quad [\text{as } e = A]$$

$$P = \left[\frac{A}{T_i}\right] t$$

Important pts. :-

→ The rate of change of integral controller o/p can be reset by changing the value of "T_i", hence this controller is also known as "Reset Controller".

→ The Disadvantage of this controller is its response to error is "slow or sluggish".

→ Derivative Controller Error :- (Rate Controller)

$$P \propto \frac{de}{dt}$$

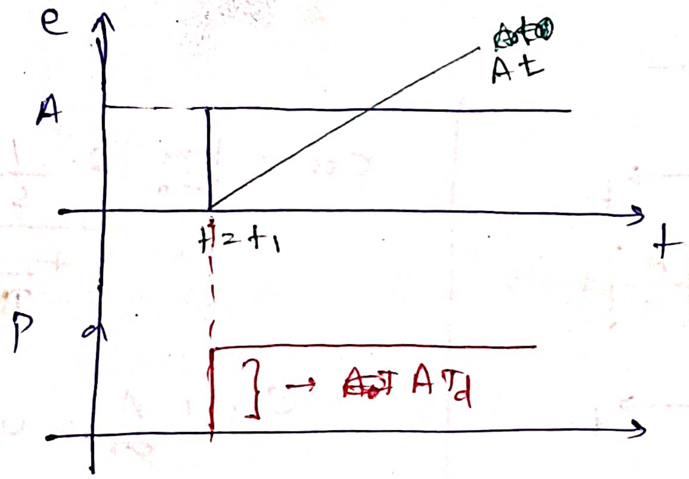
$$\Rightarrow P = K_D \frac{de}{dt}$$

Defining Derivative time / Rate time :-

$$T_d = K_D$$

$$\Rightarrow P = T_d \frac{de}{dt}$$

$$\boxed{P(s) = T_d\, s\, E(s)} \Rightarrow E(s) \rightarrow \boxed{T_d s} \rightarrow P(s)$$



For sudden error

$$P = T_d \frac{d(A)}{dt} \quad [\text{as } e = A]$$

$$P = 0$$

means, controller not responding

~~for Ramp error~~

For Ramp error :-

$$P = T_d \frac{d}{dt} A(t) \Rightarrow A T_d.$$

$$e_{ss} = \lim_{s \to 0} \frac{s \frac{A}{s^2}}{1 + \frac{A T_d s}{1 + Ts}} = \frac{A}{0} = \infty$$

-: Imp. Points :.

→ The disadvantage of this controller action is its doesn't respond to "sudden error" and for other types of errors, its response is unusually fast.

→ It is also called as "Anticipatory Control Action", because the controller sends a controller signal in Anticipations of the error.

→ The charateristics of derivative action can be co-related with feed forward control action.

Composite Control Actions :-

1) :- P + I Controller mode :-

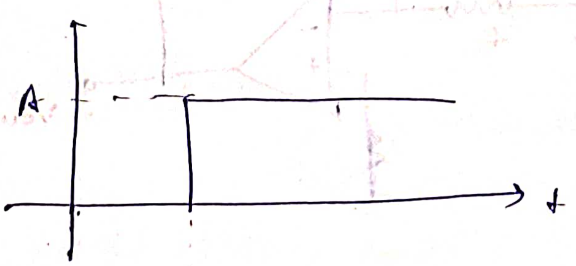
$$P = k_p e + \frac{k_p}{T_i}\int e\,dt$$

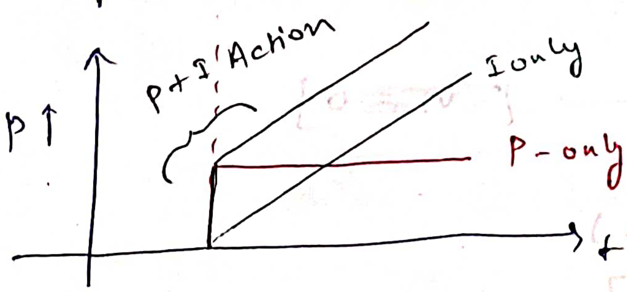
$$P(s) = \left\{k_p\left(1+\frac{1}{T_i s}\right)\right\}E(s) \Rightarrow \xrightarrow{E(s)} \boxed{k_p\left(1+\frac{1}{T_i s}\right)} \rightarrow P(s)$$

let $e = A$

$$P = k_p A + \frac{k_p}{T_i}\int A\,dt.$$

$$P = k_p A + \left[\frac{A k_p}{T_i}\right]t$$





$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{A}{s}}{1+k_p\left(1+\frac{1}{T_i s}\right)\left(\frac{1}{1+\tau s}\right)}$$

$$e_{ss} = \frac{A}{\infty} = 0.$$

⇒ It improve the Steady State response.

Now, for transient state Analysis:

For $e = \sin\omega t$

$$P = k_p \sin\omega t + \frac{k_p}{T_i}\int \sin\omega t \cdot dt$$

$$P = k_p \sin\omega t + \left(\frac{-k_p}{T_i}\right)\cos\omega t.$$

$$\text{or}\quad P = \sqrt{(k_p)^2 + \left(\frac{-k_p}{\omega T_i}\right)^2}\ \sin\left[\omega t + \tan^{-1}\left(\frac{-k_p/\omega T_i}{k_p}\right)\right]$$

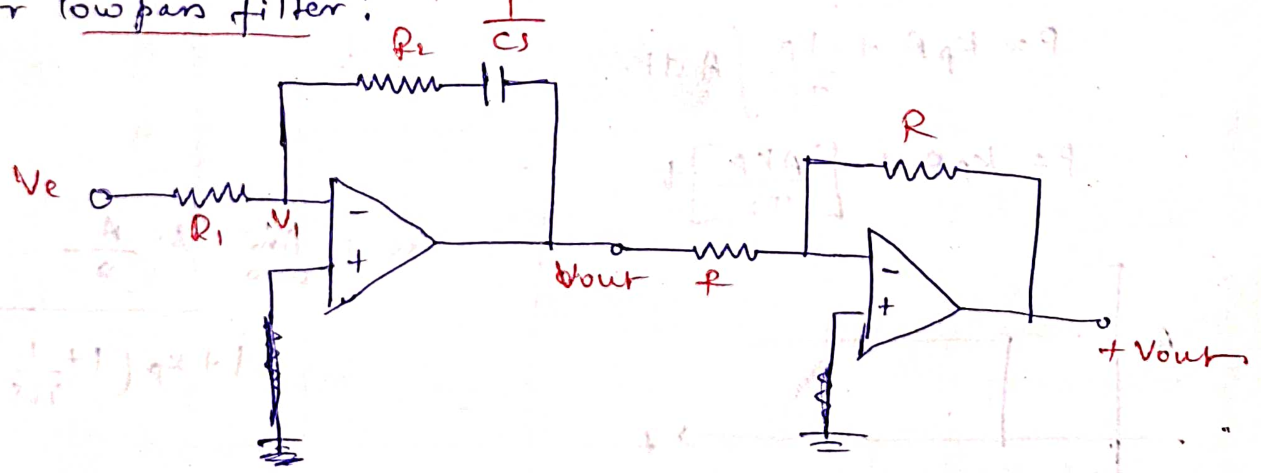
$$\text{or}\quad \boxed{P = \sqrt{(k_p)^2 + \left(\frac{k_p}{\omega T_i}\right)^2}\cdot \sin\left[\omega t - \tan^{-1}\frac{1}{\omega T_i}\right]}.$$

Important points!.

#→ This controller is ~~capable~~ capable of improving the S.S response characteristics of the system.

#→ The integral action successfully elimanates the offset of proportional action.

#→ for sinsoidal deviation the phase of the controller o/p lag by "$\tan^{-1}\left(\frac{1}{\omega T_i}\right)$". Therefore its characteristics similar to lag compensator or low pass filter.



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2CS+1}{CS}} \quad [\cancel{V_f = 0}]$$

$\boxed{V_1 = 0} \rightarrow$ (Here)

$$\therefore\quad \frac{V_e[R_2CS+1]}{R_1CS} = -V_{out}$$

$$-V_{out} = \frac{V_e R_2 \cancel{Cs}}{R_1 \cancel{Cs}} + \frac{V_e}{R_1 Cs}$$

$$+V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} \int V_e\,dt$$

Comparing this

$$P = K_p e + \frac{K_p}{T_i} \int e\,dt$$

we get similar.

→ P + D Controller mode :→

$$P = K_p e + K_p T_d \frac{de}{dt}$$

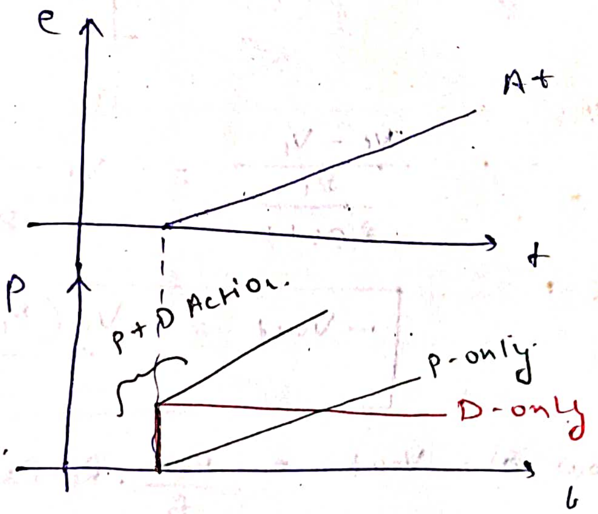
$$P(s) = \{K_p[1 + T_d s]\} E(s) \Rightarrow$$

$$E(s) \rightarrow \boxed{K_p(1+T_d s)} \rightarrow P(s)$$

let $e = At$

$$P = K_p At + K_p T_d \frac{d}{dt}(At)$$

$$\boxed{P = K_p At + A K_p T_d}$$



Steady State Analysis :

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s} \cdot \frac{A}{s^{\cancel{2}}}}{1 + K_p \frac{(1+T_d s)}{(1+Ts)}}$$

$$= \lim_{s\to 0} \frac{A}{s + s K_p \frac{(1+T_d s)}{(1+Ts)}} = \frac{A}{0} = \infty$$

[Error remains in its max value]

—! Transient State Analysis :-

let $e = \sin\omega t$

$$P = K_p \sin\omega t + K_p T_d \frac{d}{dt} \sin\omega t.$$

$$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t.$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \cdot \sin\left[\omega t + \tan^{-1}\left(\frac{\omega K_p T_d}{K_p}\right)\right]$$

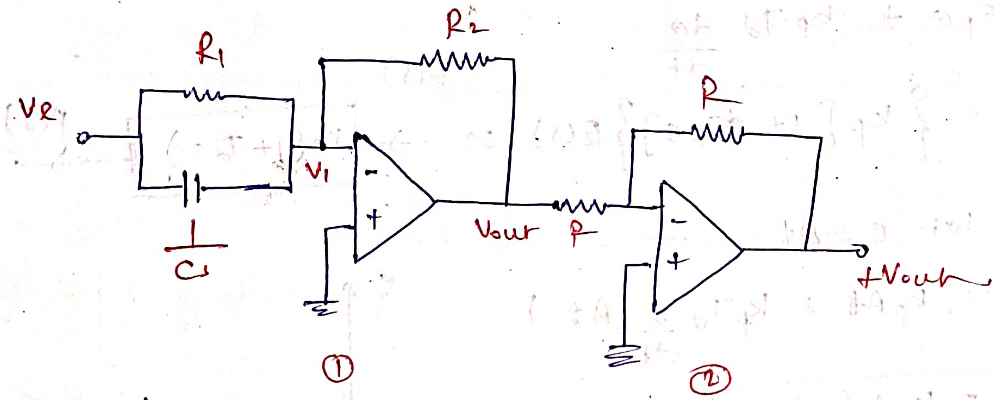
$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2} \sin\left[\omega t + \tan^{-1}\omega d\right]$$

Important points :-

#→ It is capable of improving the Transient state characteristics or speed of response of the system.

#→ It improves the stability of system in terms of speed of response.

#→ For sinusodal deviation the phase of controller o/p leads by ~~Tan⁻¹ωτd~~ "$\tan^{-1}\omega T_d$". Hence characteristics are similar to "lead compensator" or "high pass filter".



$$\frac{\dfrac{V_e - V_1}{\dfrac{R_1}{R_1Cs+1}}}{} = \frac{V_1 - V_{out}}{R_2}$$

$$\boxed{-V_{out} = \frac{V_e(R_1Cs+1)\cdot R_2}{R_1}} \quad -①$$

Now ① $-V_{out} = \frac{R_2}{R_1}\cdot R_1 Cs\, V_e + \frac{R_2}{R_1} V_e \quad \rightarrow ②$

$$+V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}R_1C\frac{dV_e}{dt}$$

$$P = K_p e + K_p T_d \frac{de}{dt}$$

Comparing these two, come out as similar.

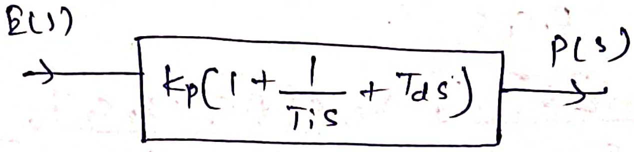
PID controller Mode :-

#→ It improves both the transient and steady state response chara. of the system.

#→ It is similar to lead lag compensator or Band stop filter.

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt + K_p T_d \frac{de}{dt}$$

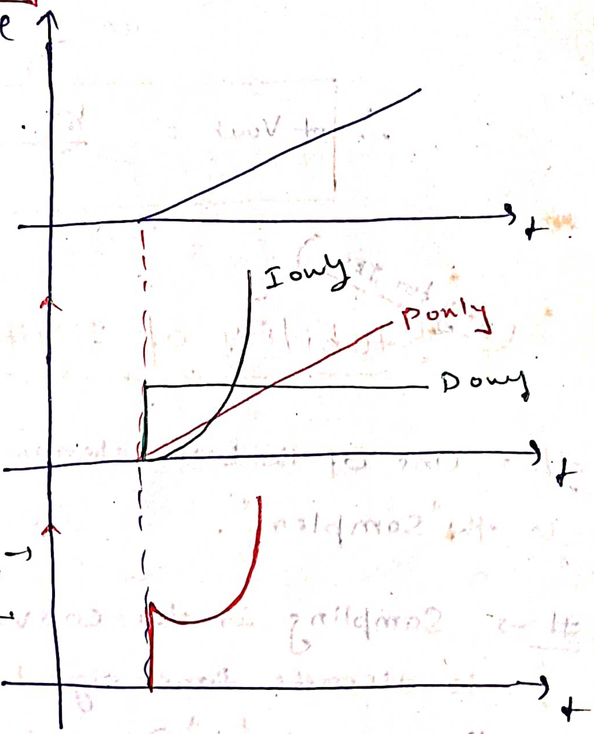
$$P(s) = \left\{ K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \right\} E(s)$$



let $e = At$

$$P = K_p At + \frac{K_p}{T_i}\int At\,dt + K_p T_d \frac{d}{dt} At$$

$$\boxed{P = (K_p A)t + \left(\frac{A K_p}{2T_i}\right)t^2 + A K_p T_d .}$$



PID Action

Now for Steady State Analysis

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^2}}{1 + K_p\left(1+\frac{1}{T_i s}+T_d s\right)\left(\frac{1}{1+Ts}\right)}$$

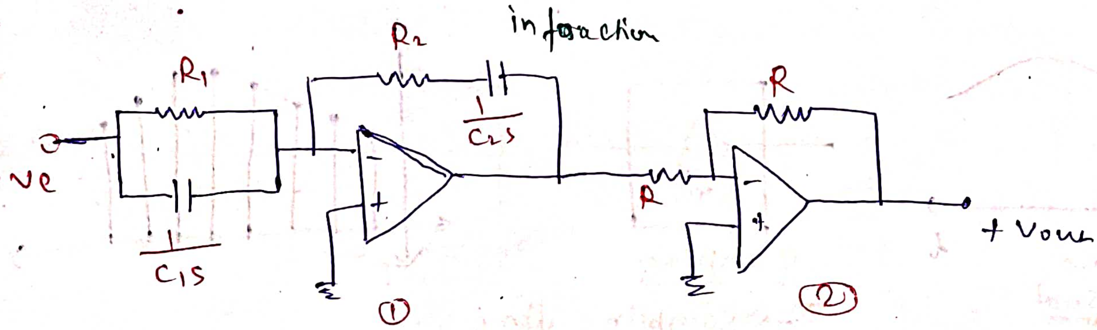
$$= \lim_{s\to 0} \frac{A}{s + \left(sK_p + \frac{sK_p}{T_i s} + T_d K_p s^2\right)\times\frac{1}{(1+Ts)}}$$

$$= \lim_{s\to 0} \frac{A}{0 + \left(0 + \frac{K_p}{T_i} + 0\right)\times 1}$$

$$= \frac{AT_i}{K_p} \approx 0 \quad [\text{as } T_i << K_p] \rightarrow \text{it is in multiple of 100.}$$

↓

in fraction



$$\frac{\dfrac{V_e - V_1}{R_1}}{R_1 C_1 s + 1} = \frac{V_1 - V_{out}}{\dfrac{R_2 C_2 s + 1}{C_2 s}} \quad \text{—— (1)}$$

— 2 —

$$-V_{out} = \frac{V_e [R_1 C_1 s + 1][R_2 C_2 s + 1]}{R_1 C_2 s}$$

$$-V_{out} = \frac{V_e [R_1 C_1 R_2 C_2 s^2 + s[R_1 C_1 + R_2 C_2] + 1]}{R_1 C_2 s}$$

$$-V_{out} = \frac{V_e \cdot \cancel{R_1} R_2 C_2 \cancel{C_2} s^{\cancel{2}}}{\cancel{R_1} \cancel{C_2} \cancel{s}} + \frac{V_e \cancel{s} [R_1 C_1 + R_2 C_2]}{R_1 C_2 \cancel{s}} + \frac{V_e}{R_1 C_2 s}$$

$$-V_{out} = V_e \left[\frac{\cancel{R_1} C_1}{\cancel{R_1} C_2} + \frac{R_2 \cancel{C_2}}{R_1 \cancel{C_2}} \right] + \frac{V_e}{R_1 C_2 s} + R_2 C_1 s V_e$$

$$-V_{out} = V_e \left[\frac{C_1}{C_2} + \frac{R_2}{R_1} \right] + \frac{V_e}{R_1 C_2 s} + R_2 C_1 s V_e$$

as $\frac{C_1}{C_2} << \frac{R_2}{R_1}$

$$\therefore \boxed{+V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C_2} \int V_e \, dt + \frac{R_2}{R_1} \cdot R_1 C_1 \frac{dV_e}{dt}}$$