Part :- 2

Time Domain Analysis :-

It is divided into two parts :-

i) Transient state ~~Ay~~ Analysis :- It deals with the nature of ~~system~~ response of system ; when subjected to any i/p.

ii) :- Steady state Analysis :- It deals with the estimation of magnitude of steady state error b/w i/p and o/p.

Standard Test Signals :-

The various i/ps or distrubances effecting the performance of a system are mathematically represented as standard test signals.

i) Sudden input → Step signal

ii) Velocity input → Ramp signal.

iii) Acceleration type input → Parabolic Signal } T.D Analysis

iv) Sudden Shocks → Impulse signals } → Stability Analysis.

Signals ① & ④ ⇒ ~~Bouded~~ Bounded Inputs

Signals ② & ③ → Un-bounded inputs.

~~Time~~ Type and the order of the Systems :-

#→ Every transfer function describing the control system is of particular type and order.

#→ The steady state analysis depends on type of the system.

#→ The type of the system is determine from open loop transfer function $G(s) \cdot H(s)$

#→ The no. of open loop poles occurring at origin determines the type of the system.

Let:

$$G(s) \cdot H(s) = \frac{K(1 + T_a s) \cdots}{s^P (1 + T_1 s) \cdots}$$

if $P = 0$ ⇒ Type 0 system

$P = 1$ ⇒ Type I system

⋮

$P = n \Rightarrow$ Type $\rightarrow$ n system.

$\rightarrow$ The transient state Analysis depends on the order of the system

$\rightarrow$ The order of the system is determine from closed loop transfer function.

$$\frac{G(s)}{1 + G(s)H(s)}$$

$\rightarrow$ Highest power of charatersitics eqn

$$1 + G(s)H(s) = 0$$

Determines the order of the system.

For Example :- The T.F of a control system is

$$\frac{C(s)}{R(s)} = \frac{1}{s+1}$$

Its type & order are

a) 0, 1

b) 1, 0

c) 1, 1

d) 1, 2

Soln.

$$\frac{G(s)}{1 + H(s)G(s)} = \frac{1}{s+1}$$

$\Rightarrow 1 + G(s)H(s) = 0$

$\Rightarrow s+1 = 0$

$\therefore \Rightarrow \boxed{\text{order} = 1}$

Now Assume $H(s) = 1$

$$\frac{G(s)}{1+G(s)} = \frac{1}{s+1}$$

$$G(s)\left[s+1-1\right] = 1$$

$$G(s) = \frac{1}{s}$$

$\therefore$ $G(s)H(s) = \frac{1}{s^1}$

$\therefore$ $\boxed{\text{Type of system is } 1.}$

Quesn: For certain unity f.d system has

$$G(s) = \frac{10(s+5)}{s^2(s+10)}$$

Its type & order is — ?

Soln

$$G(s) = \frac{10(s+5)}{s^2(s+10)}$$

$\therefore$ $\boxed{\text{Type} = 2}$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{10(s+5)}{s^2(s+10)} \cdot 1 = 0$$

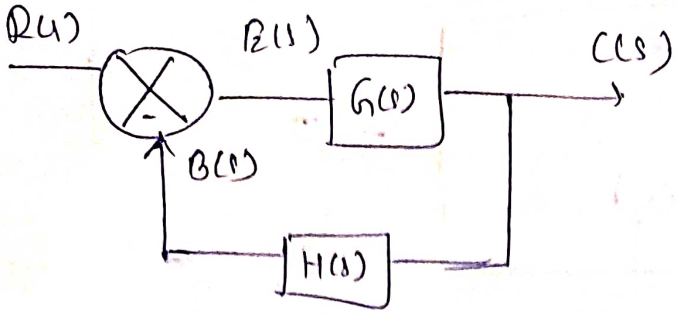
$$\frac{s^2(s+10) + 10(s+5)}{s^2(s+10)} = 0$$

$$s^3 + 10s^2 + 10s + 50 = 0$$

$\therefore$ $\boxed{\text{Order of system} = 3}$

Steady - State response analysis :-

#→ To obtain an expression for error



$$E(s) = R(s) - B(s)$$

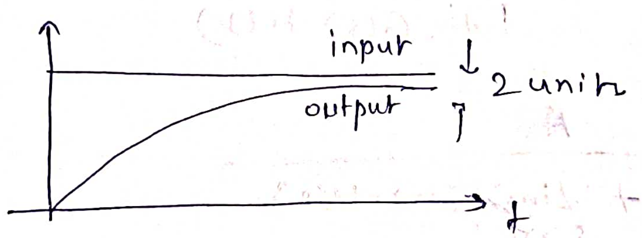
$$\Rightarrow E(s) = R(s) - C(s) \cdot H(s)$$

$$E(s) = R(s) - E(s)\, G(s) \cdot H(s)$$

$$E(s)\left[1 + G(s)H(s)\right] = R(s)$$

$$\therefore \boxed{E(s) = \frac{R(s)}{1 + G(s)H(s)}} \rightarrow \text{Error Ratio.}$$

let



$\therefore \lim_{t \to \infty} e(t) = 2$ units $(e_{ss}) \rightarrow$ magnitude of error.

So,

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

Now by Applying final value theorem :-

$$\boxed{e_{ss} = \lim_{s \to 0} s\, E(s)}$$

$$e_{ss} = \lim_{s \to 0} \left\{ \frac{s \cdot R(s)}{1 + G(s)\, H(s)} \right\}$$

$$\boxed{e_{ss} = \frac{\lim\limits_{s\to 0} s \cdot R(s)}{1 + \lim\limits_{s\to 0} G(s)\,H(s)}} \quad \text{---}(1)$$

mag. of Steady State Error

~~Sta~~ Steady State Error for different types Inputs :-

(a) Step Input :-

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \frac{\lim\limits_{s\to 0} s \cdot \frac{A}{s}}{1 + \lim\limits_{s\to 0} G(s)\,H(s)}$$

$$= \frac{A}{1 + \lim\limits_{s\to 0} G(s) \cdot H(s)}$$

K_p = Position error const. $= \lim\limits_{s\to 0} G(s)\,H(s)$

$$\therefore \boxed{e_{ss} = \frac{A}{1+K_p}}$$

(b) :- Ramp input :-

$$R(s) = \frac{A}{s^2}$$

$$\therefore e_{ss} = \frac{\lim\limits_{s\to 0} \cancel{s} \cdot \frac{A}{s^{\cancel{2}}}}{1 + G(s)\,H(s)} = \frac{A}{\lim\limits_{s\to 0} s + \lim\limits_{s\to 0} s \cdot G(s)\,H(s)}$$

$$\therefore \boxed{K_v = \text{Velocity Error Constant} = \lim_{s\to 0} s \cdot G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_v}}$$

(C) :- Parabolic i/p :.

$$R(s) = \frac{A}{s^3}$$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{\cancel{s} \cdot \frac{A}{s^{\cancel{3}\,2}}}{1 + \lim_{s\to 0} G(s)H(s)}$$

$$e_{ss} = \frac{A}{\lim_{s\to 0} s^2 + \lim_{s\to 0} s^2 G(s)H(s)}$$

$$\boxed{K_A = \text{Acceleration error Constant} = \lim_{s\to 0} s^2 G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_A}}$$

Steady state ~~Error~~ Error for different types of Systems :-

Type - 0 system

$$G(s)H(s) = \frac{K(1+T_a s)}{(1+T_1 s)}$$

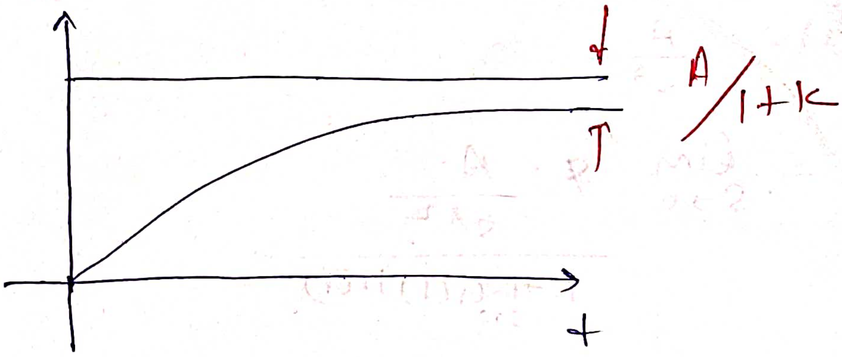
a) Step Input :-

$$R(s) = \frac{A}{s}$$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{A}{s}}{1 + \frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$e_{ss} = \frac{A}{1 + \lim_{s \to 0} \frac{K(1 + T_a s)}{(1 + T_1 s)}}$$

$$e_{ss} = \frac{A}{1 + K}$$



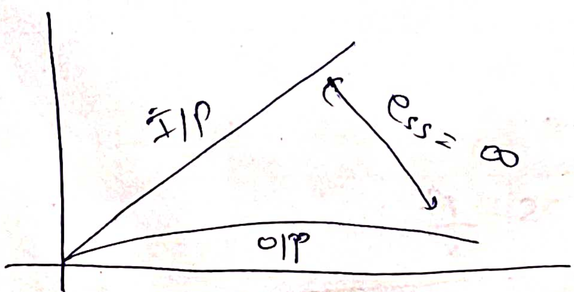
(b) Ramp Input

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{A}{s^2}}{1 + \frac{K(1 + T_a s)}{(1 + T_1 s)}}$$

$$= \frac{A}{\lim_{s \to 0} s + \lim_{s \to 0} s \cdot \frac{K(1 + T_a s)}{(1 + T_1 s)}}$$

$$e_{ss} = \frac{A}{0} = \infty$$



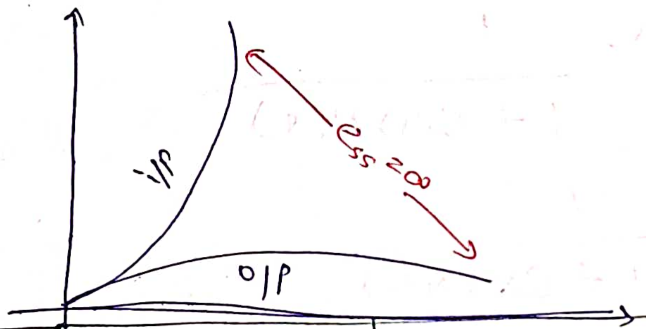
(c):- Parabolic input

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{A}{s^{\cancel{3}\,2}}}{1 + \frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$e_{ss} = \frac{A}{\lim\limits_{s \to 0} s^2 + \lim\limits_{s \to 0} s^2 \frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$e_{ss} = \frac{A}{0} = \infty$$



	Step i/p	Ramp i/p	parabolic i/p
Type - 0	$\frac{A}{1+K}$	∞	∞
Type - 1	0	A/K	∞
Type - 2	0	0	A/K

Generalised form for different i/p and different system

Observations from above :-

① :- $e_{ss} \propto \frac{1}{K}$

As $K \uparrow$ $e_{ss} \downarrow$

②:- The max. type number for linear control system is two, beyond ~~the~~ Type-2, the system exhibits non-linear characteristics, for which ~~the~~ applications of all above ~~not~~ ~~aplicable~~ concepts is not valid.

Error Series :-

$$E(s) = R(s) \cdot \frac{1}{1 + G(s) H(s)}$$

let $F(s) = \frac{1}{1 + G(s) H(s)}$

$$E(s) = R(s) \cdot F(s)$$

$$\mathcal{L}^{-1} E(s) = \mathcal{L}^{-1} R(s) \cdot F(s)$$

Dummy Variable

$$\boxed{e(t) = \int_0^\infty f(T) \cdot r(t - T)\, dT}$$

Expanding $r(t-T)$ using taylor series :-

$$r(t - T) = r(t) - T r'(t) + \frac{T^2}{2!} r''(t) - \frac{T^3}{3!} r'''(t) + \ldots$$

$$\therefore e(t) = r(t) \int_0^\infty f(T)\, dT - r'(t) \int_0^\infty T \cdot f(T)\, dT + \frac{r''(t)}{2!} \int_0^\infty T^2 f(T)\, dT - \frac{r'''(t)}{3!} \int_0^\infty T^3 f(T)\, dT + \ldots$$

Defining " Dynamic Error constants"

$$K_0 = \int_0^\infty f(T)\,dT \; ; \; K_1 = -\int_0^\infty T f(T)\,dT \; ; \; K_2 = \int_0^\infty T^2 f(T)\,dT$$

$$K_3 = -\int_0^\infty T^3 f(T)\,dT$$

$$\therefore \boxed{e(t) = K_0 r(t) + K_1 r^{\circ}(t) + \frac{K_2}{2!} r^{\circ\circ}(t) + \frac{K_3}{3!} r^{\circ\circ\circ}(t) + \cdots}$$

error series.

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

$$\mathcal{L} f(T) = F(s) = \int_0^\infty f(T) \cdot e^{-sT}\,dT$$

$$\lim_{s \to 0} F(s) = \lim_{s \to 0} \int_0^\infty f(T) \cdot e^{-sT}\,dT = \int_0^\infty f(T) \cdot dT$$

$$\therefore \boxed{K_0 = \lim_{s \to 0} F(s)}$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \int_0^\infty f(T) \cdot e^{-sT}\,dT = -\int_0^\infty T \cdot f(T) e^{-sT}\,dT$$

$$\lim_{s \to 0} \frac{d}{ds} F(s) = \lim_{s \to 0} -\int_0^\infty T \cdot f(T) \cdot e^{-sT}\,dT = -\int_0^\infty T \cdot f(T) \cdot dT$$

$$\boxed{\begin{aligned} K_1 &= \lim_{s \to 0} \frac{d}{ds} F(s) \\ K_2 &= \lim_{s \to 0} \frac{d^2 F(s)}{ds^2} \\ K_3 &= \lim_{s \to 0} \frac{d^3 F(s)}{ds^3} \end{aligned}}$$