Root Locus Technique :-

#→ Root locus is defined as the locus of closed loop poles obtain when system gain "k" is varied from '0 to ∞".

Angle and Magnitude conditions :.

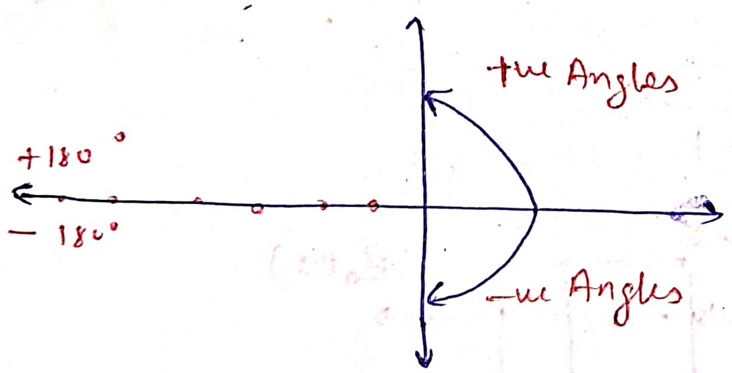
#→ The Angle conditions is used for checking whether certain points are lying on root locus or not and hence the validity of root locus for the characteristics equ. roots i.e closed loop poles.

$$1 + G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = -1$$

$$\angle G(s)H(s) = 180 - \tan^{-1}\frac{0}{1}$$

$$= 180^\circ \approx 180^\circ \approx \pm[2q+1]180^\circ$$



#→ The angle condition may be stated as for a pt to lie on root locus and angle evaluated at that pt must be an odd multiple of "±180°".

#→ The magnitude condition is used for ~~checking whether~~ finding the value of system gain "k" at any pt on root locus.

$$\Rightarrow \quad |G(s)H(s)| = \sqrt{(-1)^2 + 0^2} = 1$$

$$\boxed{|G(s)H(s)| = 1}$$

Ex1. $G(s)H(s) = \dfrac{K}{s(s+4)}$

Test whether $s = -2 + j5$ lies on Root locus or not & hence find "K".

Sol:

$$G(s)H(s)\Big|_{s=-2+j5} = \frac{K}{(-2+5j)(-2+5j+4)}$$

$$= \frac{K + 0j}{(-2+5j)(2+5j)}$$

$$= \frac{\tan^{-1}\frac{0}{K}}{\left[180^\circ - \tan^{-1}\frac{5}{2}\right]\left[\tan^{-1}\frac{5}{2}\right]}$$

$$= \frac{0^\circ}{[111.8][68.2]}$$

$$= -180^\circ$$

∴ The pt lies on root locus

Now,

$$|G(s)H(s)|_{s=-2+5j} = 1$$

$$\Rightarrow \frac{\sqrt{K^2 + 0^2}}{\sqrt{(-2)^2 + 5^2}\ \sqrt{2^2 + 5^2}} = 1$$

$$\frac{K}{\sqrt{29} \times \sqrt{29}} = 1$$

$$\Rightarrow \boxed{K = 29}$$

Construction rules of Root locus :-

Rule 1:- The root locus is symmetrical about the real axis.
$[G(s)H(s) = -1]$

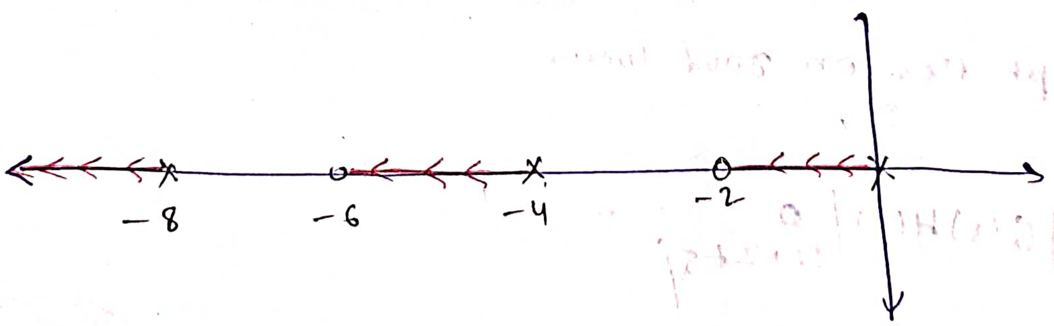
Rule 2:- Let P = no. of open loop poles.
Z = no. of open loop zeros.
& P > Z.

Then, the no. of branches of Root locus = P
" " " " " terminating at zeros = Z
" " " " " " " ∞ = P − Z

Rule :- 3:- A pt on real axis is said to be on root locus if the sum of open loop poles and zeros to the right sides of that pt. is odd.

Example :- $G(s)H(s) = \frac{K(s+2)(s+6)}{s(s+4)(s+8)}$

according to rule 2 ⇒ P = 3 ; Z = 2 ; P − Z = 3 − 2 = 1

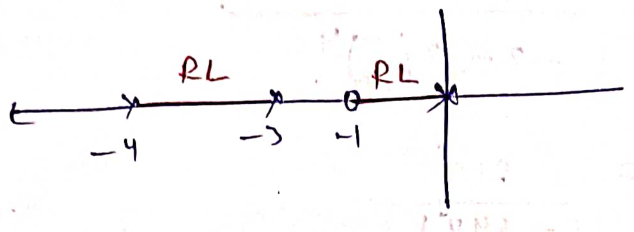


W.B

Ex 1-5

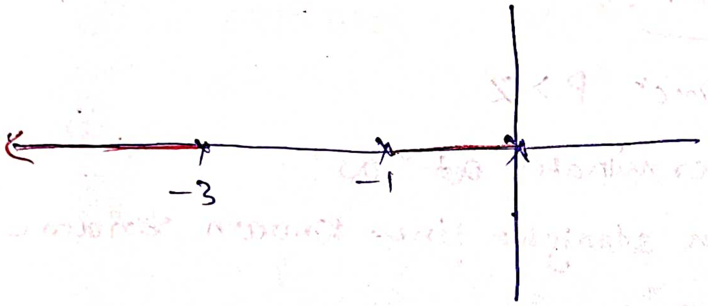
(6) :-

$$G(s)H(s) = \frac{K(s+1)}{s(s+3)(s+4)}$$



(10) :-

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$



(8) :-

$$G(s)H(s) = \frac{K}{(s+1)^4}$$

$s_1 = -3 + j4$

$$G(s)H(s)\Big|_{s=-3+j4} = \frac{K}{(-3+j4+1)^4}$$

$$= \frac{K}{(-2+j4)^4}$$

$$\Rightarrow \frac{K}{(-2+j4)^4}$$

$= -466°$.

s_1 is not lying on RL

$s_2 = -3-j2$

$$G(s)H(s)\Big|_{s=-3-j2} = \frac{K}{(-3+2j+1)^4}$$

$$= \frac{K}{(-2-2j)^4}$$

$$= \frac{0}{(-540^\circ)}$$

$$= 540^\circ.$$

Rule 4 :- Angle of Asymptotes :-

Generally in a transfer func^n $P > Z$

$\Rightarrow$ $(P-Z)$ branches ~~are~~ terminates ~~at~~ "∞".

They do so along certain straight lines ~~known~~ known as "Asymptotes of Root locus".

$\therefore$ No. of Asymptotes $= P - Z$

$$\theta = \frac{(2q+1)180^\circ}{P-Z}$$

$q = 0, 1, 2, 3 \ldots$

Ex 1. $P - Z = 2$

$$\theta_1 = \frac{[2\times(0)+1]\times 180^\circ}{2} = 90^\circ.$$

$$\theta_2 = \frac{(2\times 1+1)\times 180^\circ}{2} = 270^\circ$$

Ex 2. $P - Z = 3$

$$\theta_1 = \frac{[2\times 0+1]\times 180}{3} = 60^\circ.$$

$\theta_2 = \frac{(2\times1+1)\times180^\circ}{3} = 180^\circ$

$\theta_3 = \frac{(2\times2+1)\times180^\circ}{3} = 300^\circ.$

W.B
Ex 1-5

(17) :-

$s(s+4)(s^2+2s+2) + K(s+1) = 0$

$1 + \frac{K(s+1)}{s(s+4)(s^2+2s+5)} = 0$

$G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$

$P = 4 \;;\; Z = 1$

$P - Z = 4 - 1 = 3$

$\therefore$ No. Asympototes $= 3$

$60^\circ, 180^\circ, 300^\circ.$

Rule 5 :- Centroid :- It is the intersection pt of the Aysmptotes of root locus on "-ve" real axis. It may be or may not be a part of root locus.

$\therefore$ Centroid $= \frac{\sum \text{real part of open loop poles} - \sum \text{real part of open loop zeros}}{P - Z}$

W.B Ex:-5 (1) :-

$G(s)H(s) = \frac{K(s+6)}{(s+3)(s+5)}$

$\frac{-3+(-5) - (-6)}{1} = -8+6 = -2$

W.B Ex 1-5

(3):-

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

$$\frac{0+(-1)+(-3)-0}{3} = \frac{-4}{3} = -1.33$$

(7):-

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+5)}$$

$$\frac{0+(-1)-2-5-(-3)}{4-1}$$

$$= \frac{-8+3}{3} = -5/3$$

(14):-

$$\cancel{\frac{K(s+1)}{s^3+5s^2+(K+6)s}}$$

$$s^3+5s^2+\cancel{K}6s+K(s+1) = 0$$

$$1+\frac{K(s+1)}{s(s^2+5s+6)} = 0$$

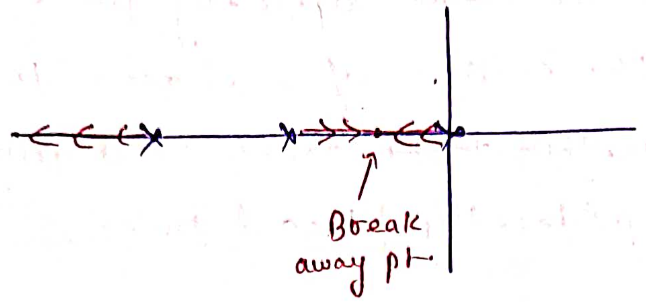
$$G(s)H(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

$$\frac{0-2-3+1}{3-1} = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

$$\text{Centroid} = (-2, 0)$$

Rule:-6: Break away points:-

Break away point is point on real axis where multiple roots of the characteristics eqn occur.



Procedure:- (find the Break away pt).

(1):- Construct $1 + G(s)H(s) = 0$

(2):- write "k" in terms of "s"

(3):- find $\frac{dk}{ds} = 0$

(4):- The roots of $\frac{dk}{ds} = 0$ will give B.A or B in points.

(5):- To test valid Break away or Break in point substitute in step 2.

If $k = +ve \rightarrow$ Valid B.A / B in point.

General predictions about Break away pts.:-

(1):- The branches of root locus either approach or leave the break away at an angle of $\pm \frac{180°}{n}$ [n = 2]

$\pm \frac{360°}{n}$ [n > 2]

where n = no. of branches approaching / leaving the break away point.

(2):- The complex conjugate path of the branches of root locus approaching or leaving the break away pt. is a circle.

(3):- Whenever there are two ~~ajan~~ adjacently placed poles on the real axis with the section of real axis b/w them as a part of root locus, then ~~there~~ there exist min one break away pt. b/w the adjacently placed poles.

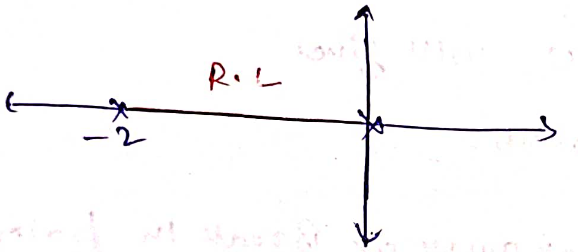
$$\cancel{G(s)H(s) = \frac{K}{s(s+2)}}$$

$$G(s)H(s) = \frac{K}{s(s+2)}$$

By rule 2:- $P = 2$; $Z = 0$

$P - Z = 2$

By rule 3:-



(4):- Asymptotes:-

$\theta_1 = 90°$; $\theta_2 = 270°$

(5):- Centroid:-

$$\frac{0 + (-2) - 0}{2} = -1$$

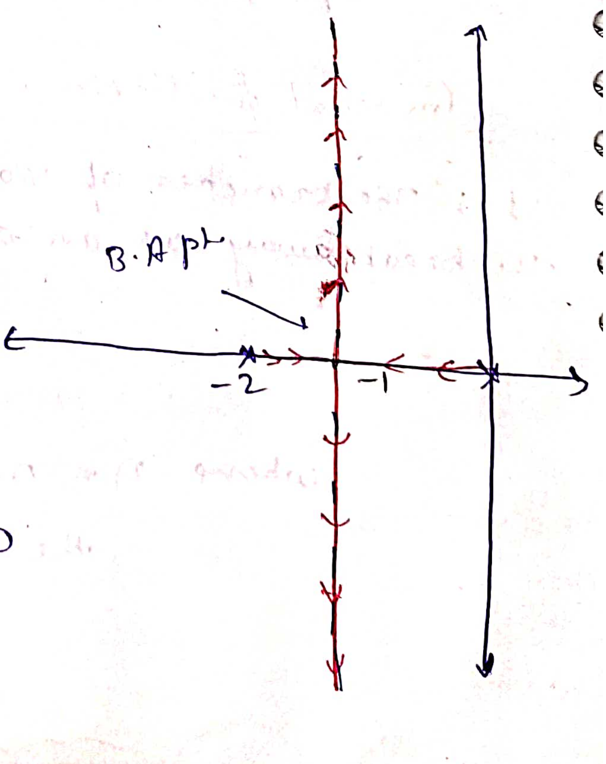
(6):- B.A. pt.:-

$s^2 + 2s + K = 0$

$K = -s^2 - 2s$

$\frac{dK}{ds} = 0 \Rightarrow 2s + 2 = 0$

$s = -1$

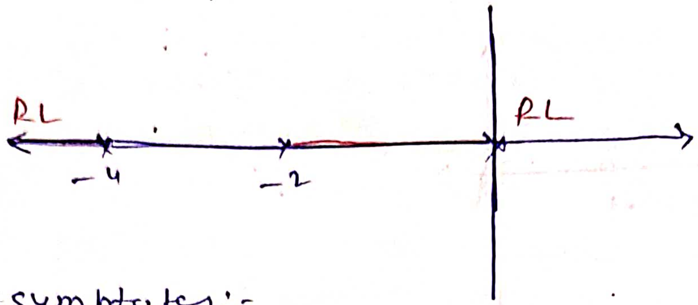


$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

Effect of adding poles to a T.F

(2):- $P = 3$; $Z = 0$; $P - Z = 3$

(3):-



(4):- Asymptotes:-

$\theta_1 = 60°$; $\theta_2 = 180°$; $\theta_3 = 300°$.

(5):- Centroid:-

$$\frac{0 + (-2) + (-4) - 0}{3} = -2$$

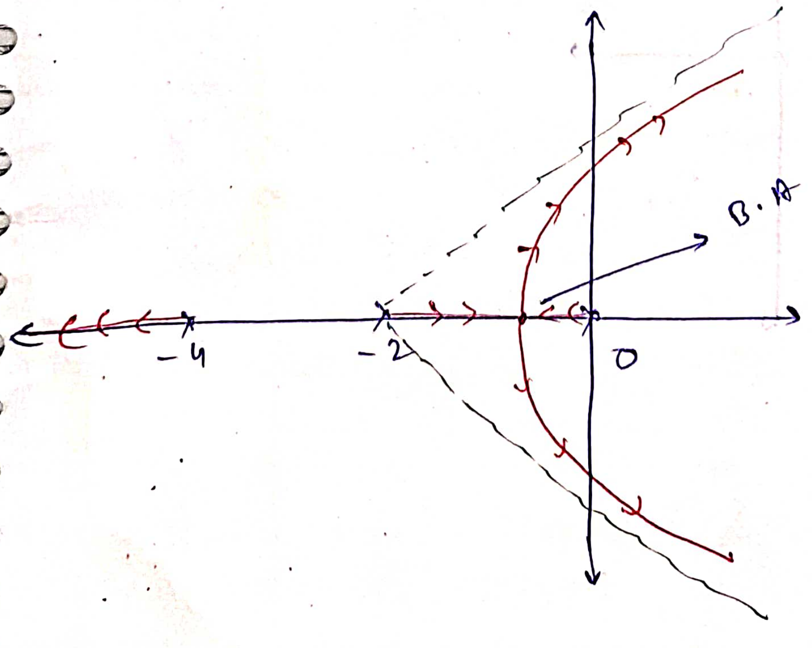
(6):- B.A pt:-

$(s^2 + 2s)(s+4) + K = 0$

$\Rightarrow K = -s^3 - 6s^2 - 8s$

$\frac{dK}{ds} = 0 \Rightarrow 3s^2 + 12s + 8 = 0$

$= -0.84, -3.15$



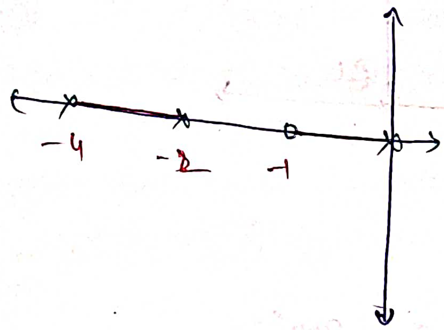
system stability decreases.

$$G(s)H(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

Effect of adding zeros to a T.F

→ $P = 3$; $Z = 1$; $P - Z = 2$

→

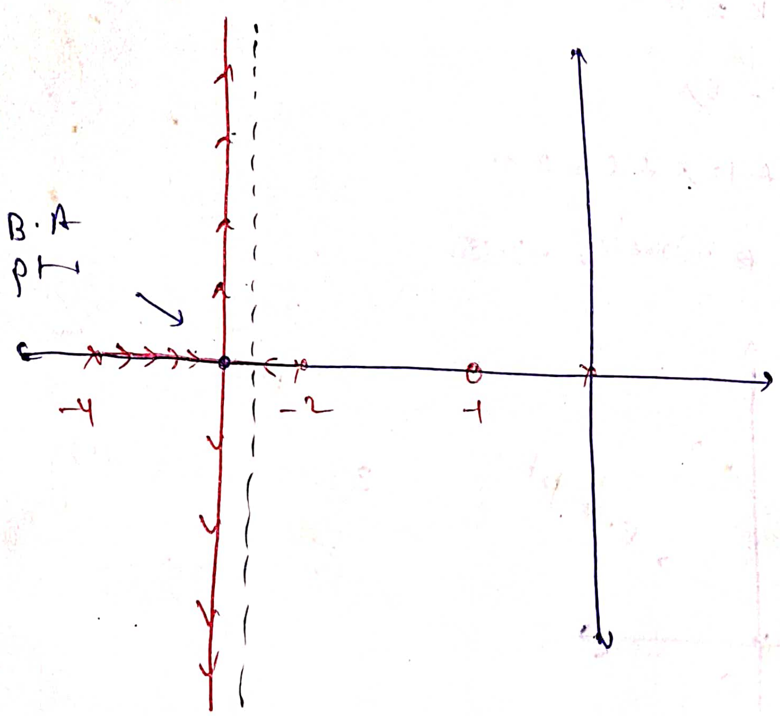


→ Asymptotes

$\theta_1 = 90°$; $\theta_2 = 270°$.

→ Centroid

$$\frac{0 + (-2) + (-4) - (-1)}{2} = -2.5$$



System stability increases.

General Predication

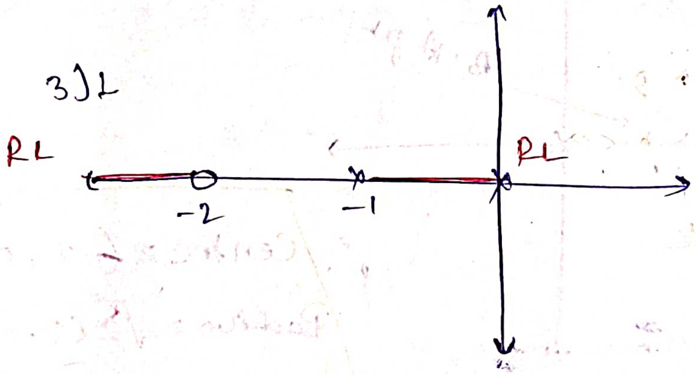
(4) :- whenever there is zero on real axis and to the left side of that zero there are no poles and zeros occurring on the real axis and the entire real axis to the left sides of that zero as a part of root locus then there exist min one B.A pt. to the left side of that zero.

Example:- $G(s)H(s) = \dfrac{K(s+2)}{s(s+1)}$

2):- $P = 2$; $Z = 1$

$P - Z = 1$

3):-



6):- B.A points

$$1 + \frac{K(s+2)}{s(s+1)} = 0$$

$$s(s+1) + K(s+2) = 0$$

$$\Rightarrow K(s+2) = -s^2 - s$$

$$K = \frac{-s^2 - s}{(s+2)}$$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{d}{ds}\,\frac{-s^2 - s}{(s+2)} = 0$$

$$\Rightarrow \frac{(s+2)(-2s-1) - (-s^2-s)\cdot 1}{(s+2)^2} = 0$$

$$-2s^2 - 4s - s - 2 + s^2 + s = 0$$

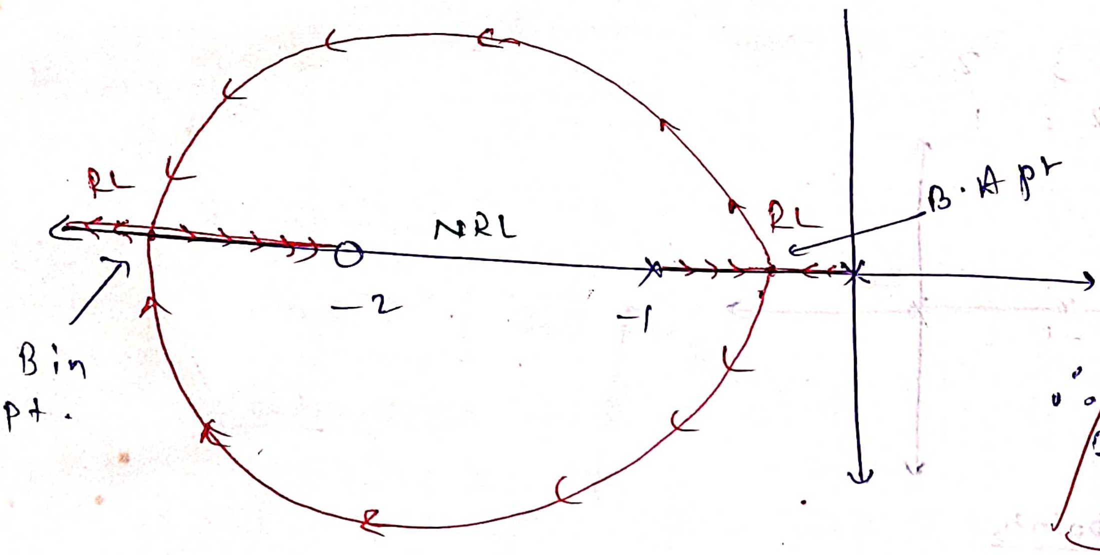
$$-s^2 - 4s - 2 = 0$$

$s^2+4s+2=0$

$$\frac{-4 \pm \sqrt{16-8}}{2}$$

$= -2 \pm \sqrt{2}$

Roots are $-2 \pm 1.4 = -0.6$
$-2-1.4 = -3.4$ } → B.A points.



∴ Centre $= (-2, 0)$
Radius $= \sqrt{2(2-1)}$
$= \sqrt{2}$

For Radius and centre of ~~this~~ circle (Root locus)

let $G(s)H(s) = \frac{K(s+b)}{s(s+a)}$

Put $s = x + jy$

$$= \frac{K[x+jy+b]}{(x+jy)(x+jy+a)}$$

$$= \frac{K[(x+b)+jy]}{x^2+jxy+ax+jxy-y^2+jay}$$

$$= \frac{K[(x+b)+jy]}{[x^2+ax-y^2]+j[2xy+ay]}$$

$\therefore \tan^{-1}\dfrac{y}{x+b} - \tan^{-1}\dfrac{2xy+ay}{x^2+ax-y^2} = 180°$

Taking Tan on b.s.

$$\frac{y}{x+b} - \frac{2xy+ay}{x^2+ax-y^2} = 0$$

$$(x^2+ax-y^2) - (2x+a)(x+b) = 0$$

$$(x^2+ax-y^2) - [2x^2+2xb+ax+ab] = 0$$

$$x^2+\cancel{ax}-y^2-2x^2-2xb-\cancel{ax}-ab = 0$$

$$\Rightarrow -x^2-y^2-2xb-ab = 0$$

$$x^2+y^2+2xb = -ab$$

$$(x+b)^2+y^2 = -ab+b^2$$

$$(x+b)^2+y^2 = b(b-a)$$

$\therefore$ $\boxed{\begin{array}{l}\text{Centre} = (-b, 0)\\ \text{Radius} = \sqrt{b(b-a)}.\end{array}}$

As in the given Example

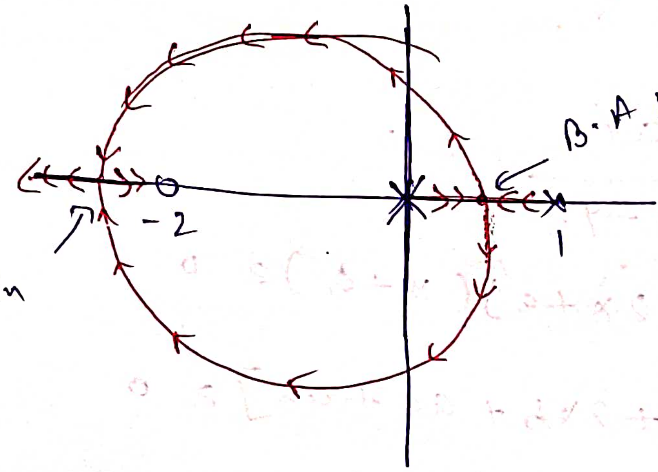
$$\text{B.A pts} = \underbrace{-2}_{\text{Centre}} \pm \underbrace{\sqrt{2}}_{\text{Radius}}$$

$\therefore$ $\boxed{\text{B.A pts} = \text{Centre} \pm \text{Radius}}$

↳ When root ~~locus~~ locus is "Circle".

Case:- 2

$$G(s)H(s) = \frac{K(s+2)}{s(s-1)} = \frac{K(s+b)}{s(s-a)}$$

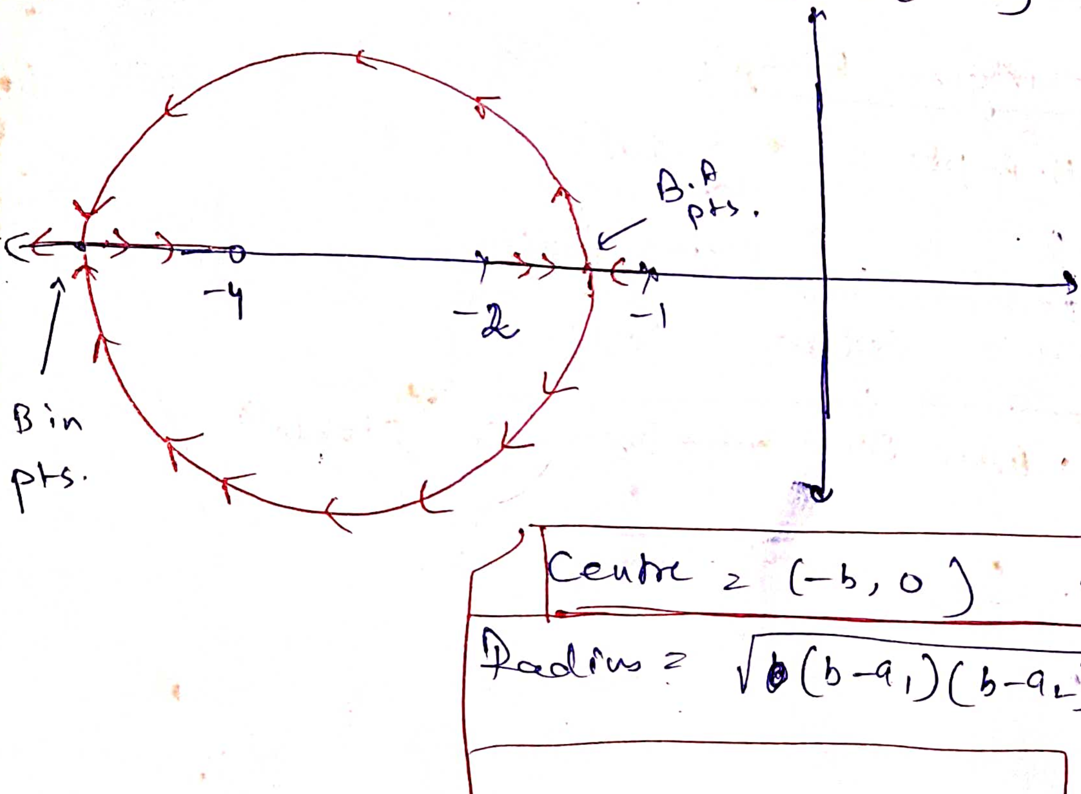


Centre $= (-b, 0) = (-2, 0)$
Radius $= \sqrt{b(b+a)} = \sqrt{2(2+1)}$ $= \sqrt{6}$

$\therefore$ B.A pts $= -2 \pm \sqrt{6}$

Case:- 3 :⇒

$$G(s)H(s) = \frac{K(s+4)}{(s+1)(s+2)} = \frac{K(s+b)}{(s+a_1)(s+a_2)}$$



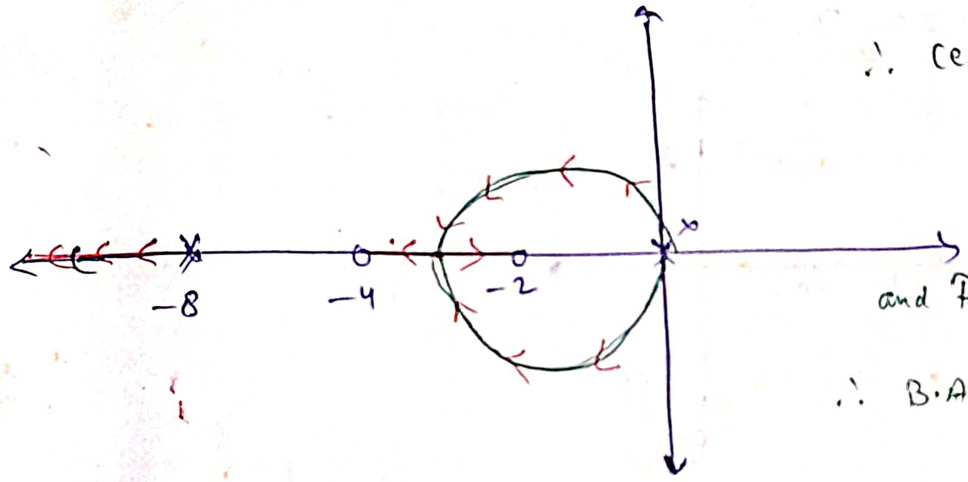
Centre $= (-b, 0) = (-4, 0)$
Radius $= \sqrt{(b-a_1)(b-a_2)} = \sqrt{(4-1)(4-2)}$ $= \sqrt{6}$

$$\boxed{B \cdot A \text{ pts} = -4 \pm \sqrt{6}}$$

General Prediacation

(5):- whenever there are two adjeecently placed zeros on the real axis with the section of real axis b/w them as root locus there exist min one breakaway pt b/w the adjeecently placed zeros.

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s^2(s+8)} = \frac{K(s+b_1)(s+b_2)}{s^2(s+a)}$$



∴ centre of this locus = $\frac{-2 \times 4}{2+4} = -\frac{8}{6}$

and Radius " " locus = $\frac{8}{6}$

∴ B·A pts = $-\frac{8}{6} \pm \frac{8}{6}$

$= 0, -\frac{8}{3}$

$$T \cdot F = \frac{K(s+b_1)(s+b_2)}{s^2}$$

Put $s = x + jy$

$$\left(x + \frac{b_1 b_2}{b_1+b_2}\right)^2 + y^2 = \left(\frac{b_1 b_2}{b_1+b_2}\right)^2$$

$$\text{Centre} = \left[-\frac{b_1 b_2}{b_1+b_2}, 0\right]$$

$$\text{Radius} = \frac{b_1 b_2}{b_1+b_2}$$

11/11/2008

Rule :- 6

Existance of Complex Break away Points :-

$$G(s)H(s) = \frac{K}{s(s^2+2s+2)}$$

Imp

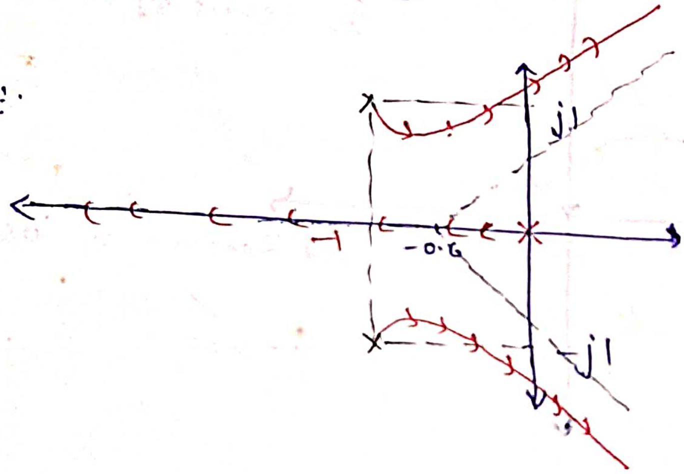
→ The complex B·A pts. are said to be valid if the real part of the complex B·A pt. and the real part of complex poles are same)

Case :- 1 :-

By rule :- 2 :- $P = 3$; $Z = 0$

$P - Z = 3$

By rule 3 :-



By rule 4 :- Aysmptotes is 3

and they are $\theta_1 = 60°$; $\theta_2 = 180°$; $\theta_3 = 300°$

By rule 5 :- Centroid

$$\frac{0 + (-1) + (-1) - 0}{3}$$

$$= -0.\dot{6}$$

By rule 6 :- $s^3 + 2s^2 + 2s + K = 0$

$$K = -s^3 - 2s^2 - 2s$$

$$\frac{dK}{ds} = 0 \Rightarrow -3s^2 - 4s - 2 = 0$$

$$\Rightarrow 3s^2 + 4s + 2 = 0$$

$\therefore$ B.A pt = roots of equ. $\frac{dk}{ds} = 0$

$$\Rightarrow \frac{-4 \pm \sqrt{16 - 4\times2\times3}}{6}$$

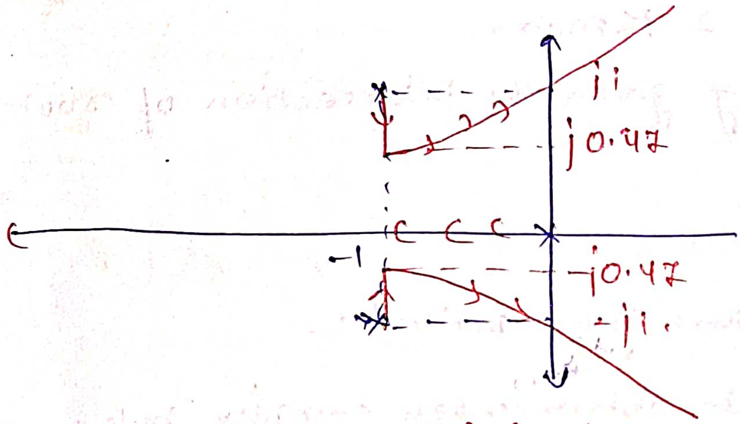
$$= -0.6 \pm j0.47,$$

(Hence, not a valid B.A point)

Now: Case:- 2:

let the B.A pt for above T.f is
$-1 \pm j0.47$.

in that case,



~~for complex roots B.A will~~

W.B:- Ex:-5

(16) :- $G(s) = \frac{K}{s(s+4)(s^2+4s+20)}$ $(0 < K < \infty)$

for B.A pt:- $(s^2+4s)(s^2+4s+20)+K=0$

$$s^4+4s^3+20s^2+4s^3+16s^2+80s+K=0$$

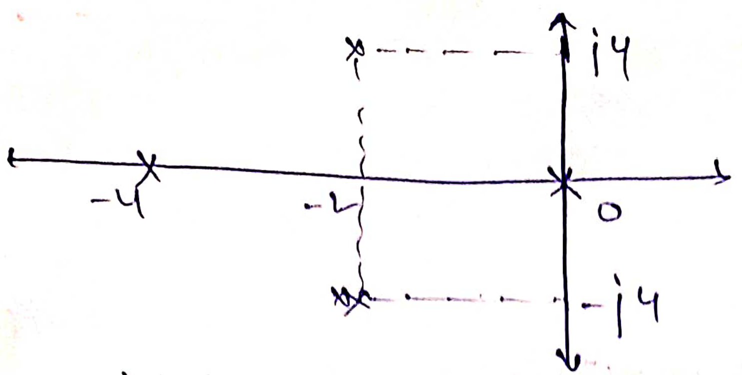
$$s^4+8s^3+36s^2+80s+K=0$$

$$K = -s^4-8s^3-36s^2-80s$$

$$\frac{dK}{ds}=0 \Rightarrow 4s^3+24s^2+72s+80=0$$

$$\Rightarrow s^3+6s^2+18s+20=0$$

$\therefore$ B.A pts are :- $-2,\ -2\pm j2.5$.



∵ All the B.A pts are valid.
so.

Rule :- 7 :- Intersection of root locus with img. axis :-
Imp

The roots of the auxillary eqn.

A(s) to $k = K_{max}$

from Routh Array gives the intersection of root locus with img. axis.

Rule :- 8 :- Angle of Departure and arrival :-

→ The angle Departure is obtain when complex poles terminate at ∞.

$$\phi_D = 180° + \phi$$

$$\boxed{\text{where } \phi = \Sigma\phi_z - \Sigma\phi_p}$$

→ The angle of arrival is obtain at complex zeros.

$$\phi_A = 180° - \phi$$

$$\boxed{\text{where } \phi = \Sigma\phi_z - \Sigma\phi_p}$$

Ex:- $G(s)H(s) = \dfrac{K}{s(s+2)(s+4)}$

7) I - $s^3 + 6s^2 + 8s + k = 0$

s^3	1	8
s^2	6	K
s^1	$\frac{48-k}{6}$	
s^0	K	

i) $\dfrac{48-k}{6} > 0$

$\Rightarrow \cancel{k} < 48$

ii) $k > 0$

$\Rightarrow \boxed{0 < k < 48}$

$\therefore K_{marginal} = 48$

$A(s) = 6s^2 + k = 0$

$6s^2 + 48 = 0$

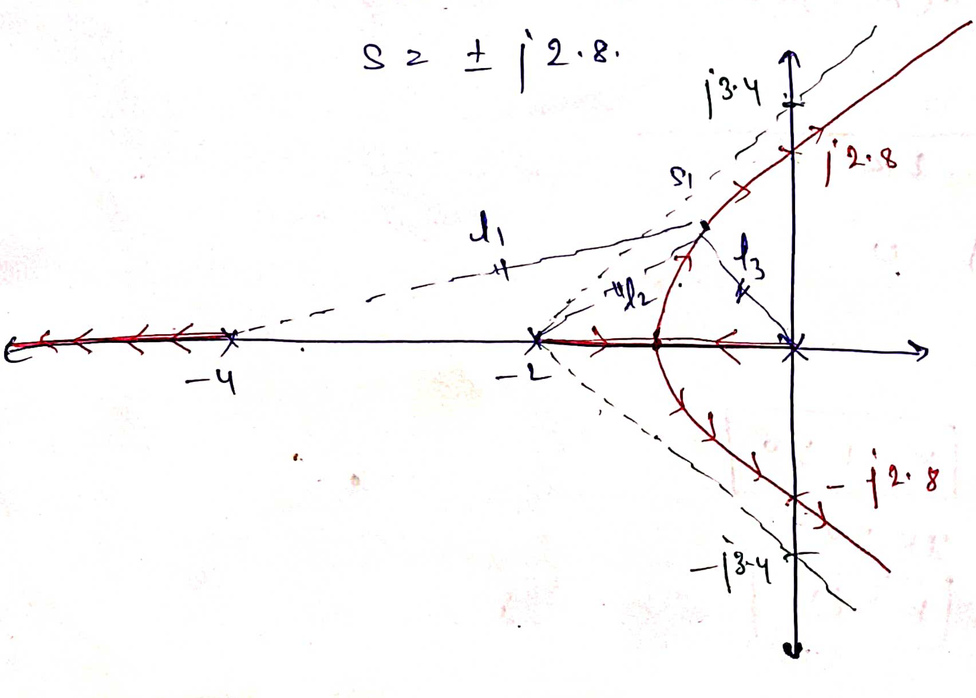
$s = \pm j2.8.$

The ~~inpe~~ intersection pt. of Asymptotes are

$\theta = \tan^{-1} \dfrac{y}{0-(-2)}$

$\tan 60 = \dfrac{y}{2}$

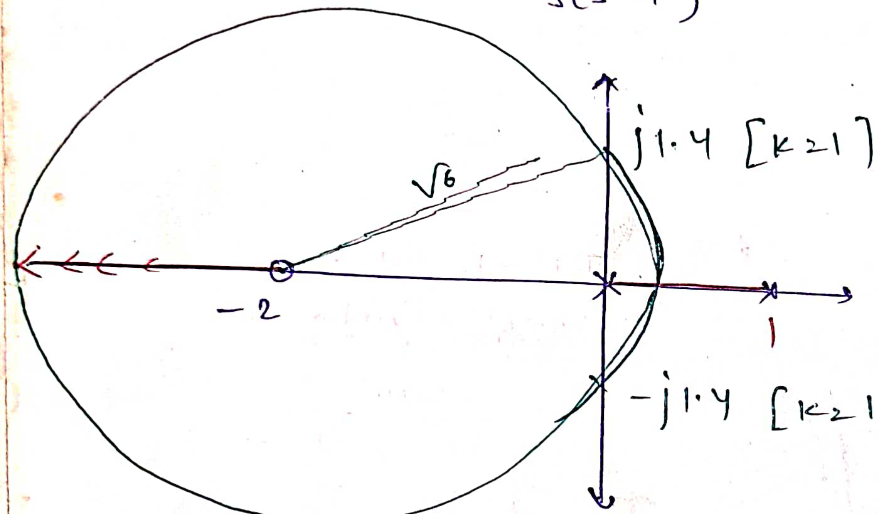
$$\boxed{\begin{aligned} y &= \tan 60 \times 2 \\ &= \sqrt{3} \times 2 = 3.4. \end{aligned}}$$



$$\boxed{K = \frac{\text{Product of vector lengths of Poles}}{\text{Product of vector lengths of Zeros}}}$$ → Generalized formula

$\boxed{K = l_1 \times l_2 \times l_3}$ → for this Example.

Example 1:- $G(s) = \dfrac{K(s+2)}{s(s-1)}$



∴ $\boxed{B.A\,pt = -2 \pm \sqrt{6}}$

$1 + G(s)H(s) = 0$

$\Rightarrow s^2 - s + Ks + 2K = 0$

$\Rightarrow s^2 + s(K-1) + 2K = 0$

s^2	1	2K
s^1	(K−1)	0
s^0	2K	

∴ i) $\boxed{K - 1 > 0}$

ii) $2K > 0$

$\Rightarrow \boxed{K > 0}$

$\therefore$ $K_{mar} = 1$

$\therefore$ $A(s) = s^2 + 2k = 0$

$s^2 + 2 = 0$

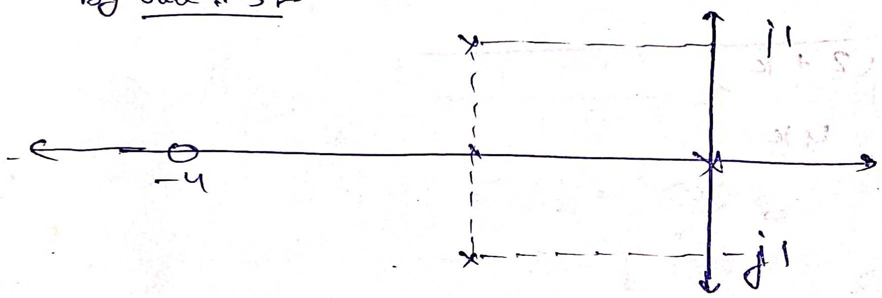
$s = \pm j1.4.$

<u>Example:-</u> $G(s) = \dfrac{k(s+4)}{s(s^2+2s+2)}$

By <u>rule 1-2:-</u> $P = 3$; $Z = 1$

$P - Z = 3 - 1 = 2$

By <u>rule 3:-</u>



4) :- $\theta_1 = 90°$; $\theta_2 = 270°$

5) Centroid $= \dfrac{0 + (-1)(-1) - (-4)}{2}$

$= +1.$

6) :- <u>BA points</u>

$s^3 + 2s^2 + 2s + k(s+4) = 0$

$k(s+4) = -s^3 - 2s^2 - 2s$

$k = \dfrac{-s^3 - 2s^2 - 2s}{(s+4)}$

$\dfrac{dk}{ds} = 0 \Rightarrow \dfrac{(s+4)(-3s^2 - 4s - 2) - (-s^3 - 2s^2 - 2s)\cdot 1}{(s+4)^2} = 0$

$\Rightarrow \; -3s^3 - 4s^2 - 2s - 12s^2 - 16s - 8 + s^3 + 2s^2 + 2s = 0$

$$-2s^3 - 14s^2 - 16s - 8 = 0$$

$$s^3 + 7s^2 + 8s + 4 = 0$$

$\boxed{\text{BA pts} = -5.7 \; ; \; -0.6 \pm j0.47}$ (not valid)

By Rule 7 :-

$$s^3 + 2s^2 + 2s + K(s+4) = 0$$

$$s^3 + 2s^2 + 2s + Ks + 4K = 0$$

$$s^3 + 2s^2 + s(2+K) + 4K = 0$$

s^3	1	$(2+K)$
s^2	2	$4K$
s^1	$\frac{4-2K}{2}$	0
s^0	$4K$	

$$\frac{4-2K}{2} > 0$$

$\Rightarrow \; 4 > 2K$

$\Rightarrow \; K < 2.$

$\therefore \; K_{mar} = 2$

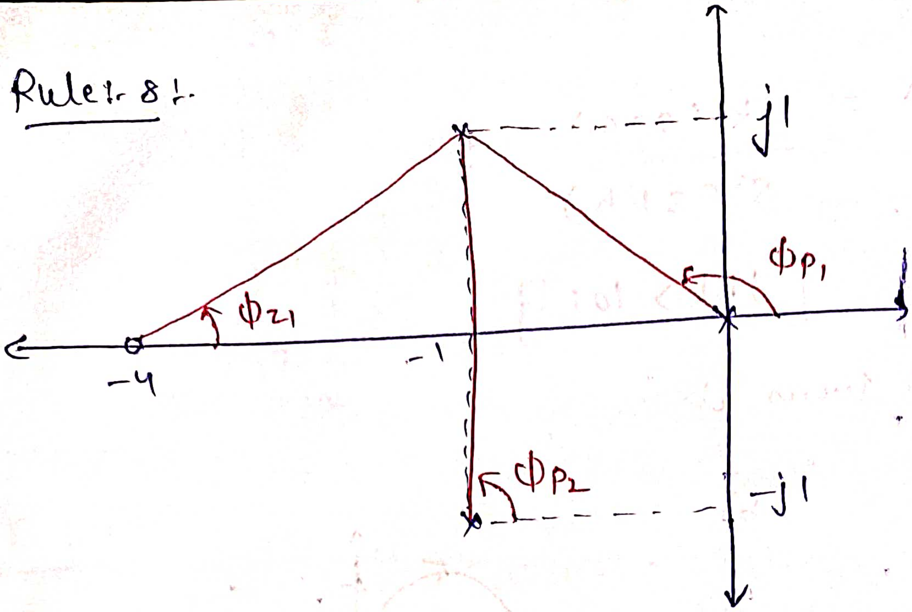
$\therefore \; A(s) = 2s^2 + 4K = 0$

$$2s^2 + 4 \times 2 = 0$$

$$s^2 = -4$$

$$s = \pm j2$$

Rule :- 8 :-



$$\phi_{p_1} = 180^\circ - \tan^{-1}\frac{(1-0)}{0-(-1)} = 180^\circ - 45 = 135^\circ$$

$$\phi_{p_2} = 90^\circ$$

$$\phi_{z_1} = \tan^{-1}\frac{(1-0)}{-1-(-4)} = \tan^{-1}\left(\frac{1}{3}\right) = 18^\circ$$

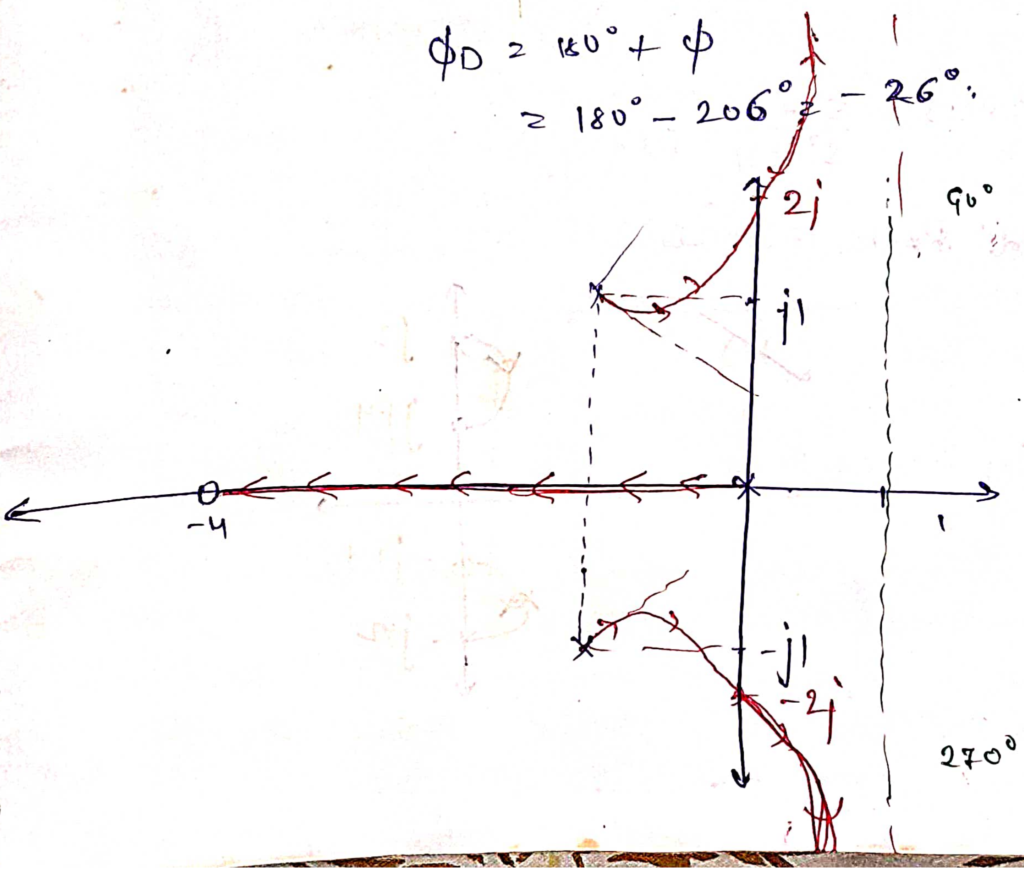
$$\therefore \phi = \Sigma\phi_z - \Sigma\phi_p$$

$$= 18 - [135 + 90^\circ]$$

$$= -206^\circ.$$

$$\phi_D = 180^\circ + \phi$$

$$= 180^\circ - 206^\circ = -26^\circ.$$



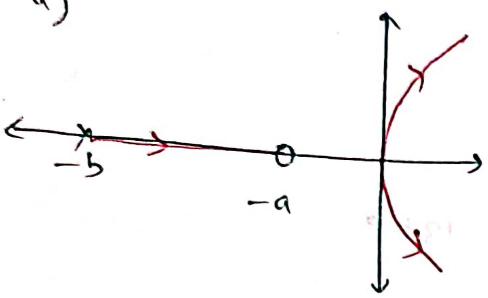
Example :- ①

$$G(s)H(s) = \frac{K(s+a)}{s^2(s+b)}$$

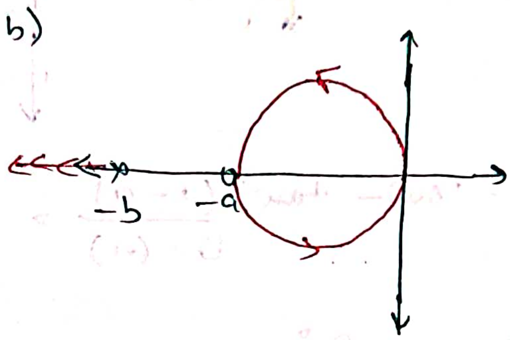
$$[\,|b| > |a|\,]$$

The valid root locus is

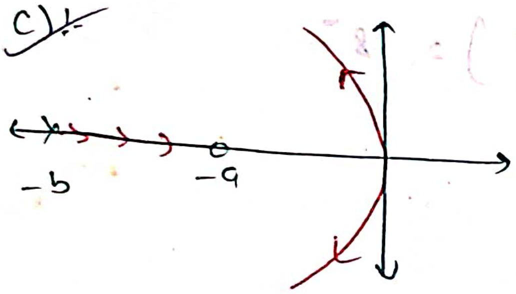
a)



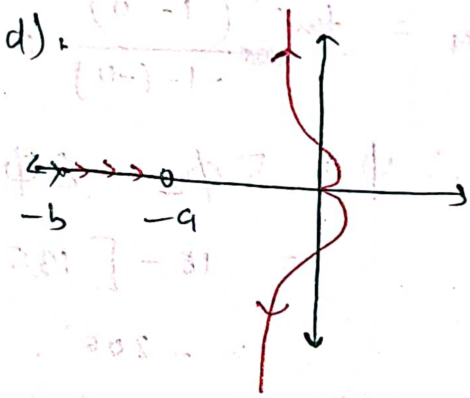
b)



c) ✓



d)

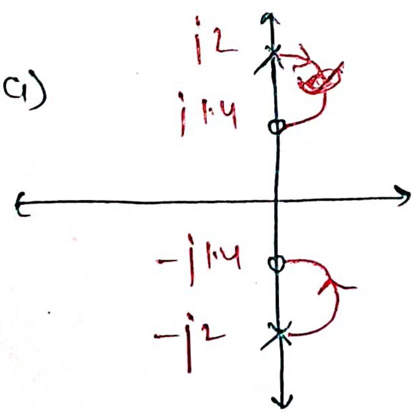


Example :- ②

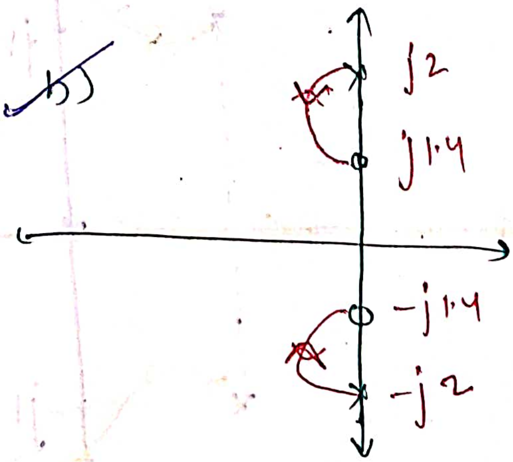
$$G(s)H(s) = \frac{s^2+2}{s^2+4}$$

The valid Root loci is

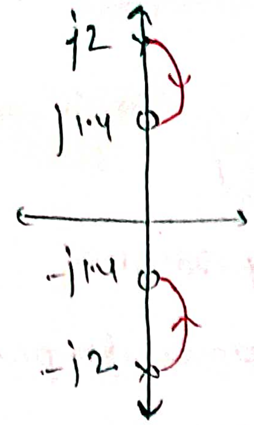
a)



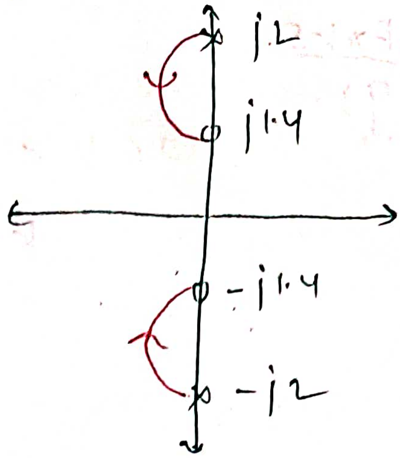
b) ✓



c)



d)



Ex-1

let $G(s)H(s) = \frac{K(s+1)}{s^2(s+2)}$

$s^3 + 2s^2 + Ks + K = 0$

s^3	1	K
s^2	2	K
s^1	K/2	0
s^0	K	

$\boxed{K > 0}$

$K_{mar} = 0$

Ex-2

$s^2 + 4 + s^2 + 2$

$2s^2 + 6 = 0$

s^2	2
s^1	4
s^0	6

W.B. Ex 1-5

Ques 9:

$0 \le K < 1 \Rightarrow$ overdamped

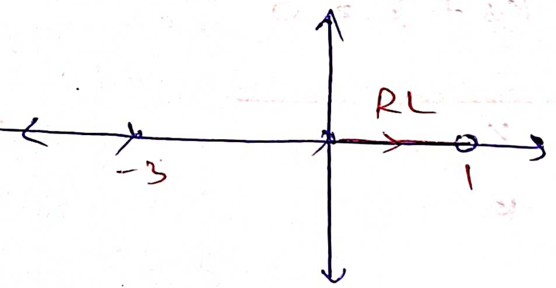
$K = 1 \Rightarrow$ critically damped

$1 < K < 5$ = Underdamped

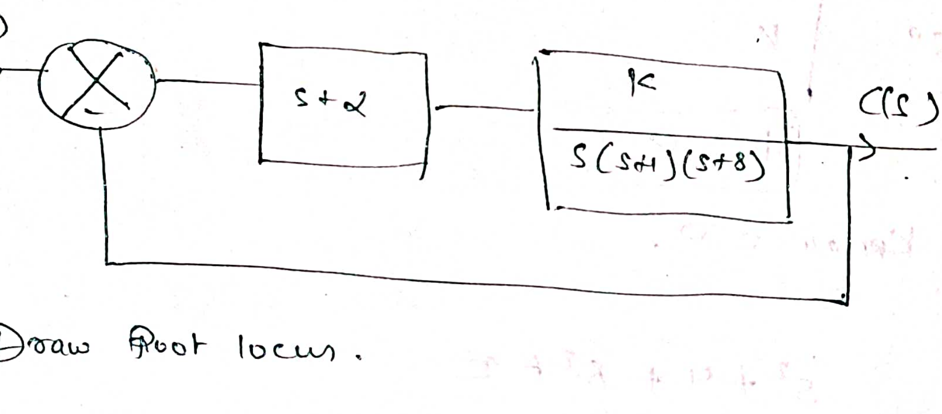
$K > 5$ = overdamped

Ques 12:

$$G(s) = \frac{K(1-s)}{s(s+3)}$$



Ques



Draw Root locus.

Sol:

$$G(s)H(s) = \frac{K(s+\alpha)}{s(s+1)(s+8)}$$

Root locus can't be drawn because the location of zero is not specify.

Root Contours:

Root Contours are multiple locus diagrams obtain by

Varying multiple parameters in a T.F. Drawn on same S-plane.

Case :- 1 :- for $\alpha = 0$

$$G(s)H(s) = \frac{K \cdot \cancel{s}}{\cancel{s}(s+1)(s+8)} = \frac{K}{(s+1)(s+8)}$$

Case :- 2 :-

$$s(s+1)(s+8) + Ks + K\alpha = 0$$

$$1 + \frac{K\alpha}{s(s+1)(s+8) + Ks} = 0$$

$$1 + G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = \frac{K\alpha}{s(s+1)(s+8) + Ks}$$

let $\boxed{K = 1}$

$$G(s)H(s) = \frac{\alpha}{s(s+1)(s+8) + s}$$

$$G(s)H(s) = \frac{\alpha}{s[s^2 + 9s + 9)}$$

Root Sensitivity :-

→ let the variable that change its value $= \alpha$

→ let the parameter that changes the value of $\alpha = \beta$

$$S_{\beta}^{\alpha} = \frac{\frac{\partial \alpha}{\alpha}}{\frac{\partial \beta}{\beta}} = \frac{\%\text{ Change in } \alpha}{\%\text{ Change in } \beta}$$