same procedure, In this case, we check the sign change by taking lim $\epsilon \to 0$, for expression carrying "ϵ".

$$\lim_{\epsilon \to 0} \frac{-4\epsilon - 4 - 5\epsilon^2}{2\epsilon + 2} = \frac{-4}{2} = -2$$

∴ There are two sign changes

$+\infty \to -2$

then $-2 \to 5$

∴ ⇒ Two C.L. poles are lying in RHS of s-plane (un-stables)

W.B
Q. 4-4

Ques 3:-

$P(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0 (8)	0 (24)	0	0
s^2	6	16	0	0
s^1	2.6	0	0	0
s^0	16	0	0	0

$\frac{16-12}{2} = \frac{4}{2}$ $= \frac{40-16}{2}$ $= 20 - 8$

$\frac{32-0}{2} = $

For this case we make a auxillary eqn by taking coefficient of rows above "0" rows. And the coefficient of auxillary eqn become the coefficient zero row, then again we carry out the procedure.

∴ → Auxillary Equn.

$A(s) = 2s^4 + 12s^2 + 16$ → (differentiate the Auxillary equn)

∴ $\frac{d}{ds} A(s) = 8s^3 + 24s$

#→ Even this case also system may be stable, marginally stable or Unstable.

To check this

as the roots of A(s) = C.L poles lying on jω axis,

∴ Put $s^2 = y$

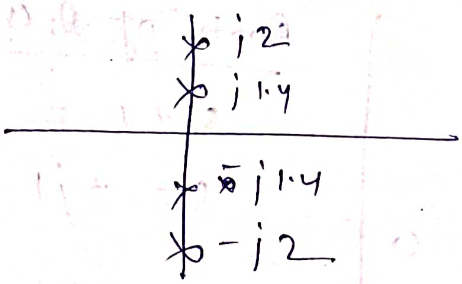
$2y^2 + 12y + 16 = 0$

∴ roots = −2, −4.

∴ $(s^2+2)(s^2+4) = 0$

$s = \pm j\sqrt{2} = \pm j1.4$

$s = \pm j\sqrt{4} = \pm j2$



#→ If the roots of the Auxillary equn lies on different locations of img axis, then the system is said to be marginally stable.

Marginally stable

(14) :- $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$.

s^5	2	4	2
s^4	1	2	1
s^3	0 (4)	0 (4)	0
s^2	1	1	0
s^1	0 (2)	0	0
s^0	1		0

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds} A_1(s) = 4s^3 + 4s$

$A_2(s) = s^2 + 1$

$\frac{d}{ds} A_2(s) = 2s$

Roots of $A_1(s)$

Put $s^2 = y$

$y^2 + 2y + 1 = 0$

$\frac{-2 \pm \sqrt{4-2}}{2}$

$= -1, -1$

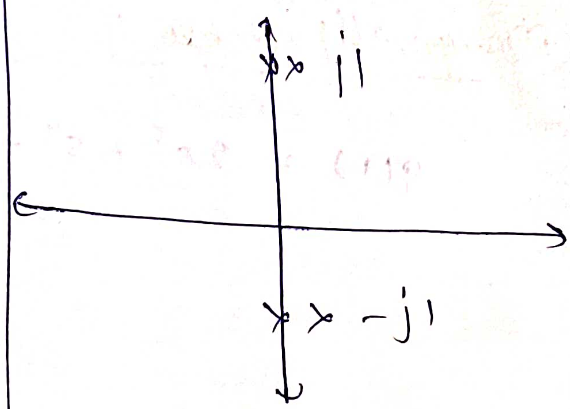
$(y+1)^2 = 0$

$(s^2+1)^2 = 0$

Roots of $A_2(s)$

$s^2 + 1 = 0$

$s = \pm j1$



#. As the roots are repeating for $A_1(s)$,

so even the I^{st} coloumn is "+ve" the system is unstable. with ~~two~~ four roots of img. axis and one on L.H.S.

(16) :-

$P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15.$

s^5	1	2	3
s^4	1	2	15
s^3	~~0~~ ϵ	-12	0
s^2	$\frac{-12-2\epsilon}{\epsilon} = -\infty$	15ϵ	0
s^1	$\frac{144\epsilon + 9\epsilon^2}{-12-2\epsilon} = 0$	0	0
s^0	15	0	0

$\frac{2-2}{1}$

$\frac{3-15}{1} = $

$\frac{-12-2\epsilon}{\epsilon}$

$\frac{15\epsilon - 0}{\epsilon}$

$\frac{144 + 24\epsilon - 15\epsilon}{\frac{-12-2\epsilon}{\epsilon}}$

$\frac{(144 + 9\epsilon)\epsilon}{-12-2\epsilon}$

To check sign change

$$\lim_{\epsilon \to 0} \frac{-12-2\epsilon}{\epsilon} = -\infty$$

$$\lim_{\epsilon \to 0} \frac{144\epsilon + \epsilon^2}{-12-2\epsilon} = 0$$

(17) :- O.L.T.F

$$\frac{1}{s^3 + 1.5s^2 + s - 1}$$

s^3	1	1
s^2	1.5	-1
s^1	1.6	0
s^0	-1 → sign change	

∴ open loop transfer func unstable

Now, For closed loop T.F

~~1 + G(s) H(s) = 0~~

$$G(s)H(s) = \frac{20(s+1)}{s^3 + 1.5s^2 + s - 1}$$

$$1 + G(s)H(s) = 0$$

~~$s^3 + 1.5s^2 + s - 1$~~

$$1 + \frac{20(s+1)}{s^3 + 1.5s^2 + s - 1} = 0$$

$$s^3 + 1.5s^2 + s - 1 + 20s + 20 = 0$$

$$s^3 + 1.5s^2 + 21s + 19 = 0$$

s^3	1	21
s^2	1.5	19
s^1	8.33	
s^0	19	

Closed loop Absolutely stable

10/11/2008

Relative Stability Analysis using Routh Array :-

#→ If the closed loop pole move away from img. axis in the L.H.S of s-plane then the time taken by the response is reaching the steady state value of i/p is less and system is said to be relatively more stable.

#→ To Check the relative stability of the characteristics eqn roots using routh Array, the origin of s plane is shifted more -vely as shown in fig.

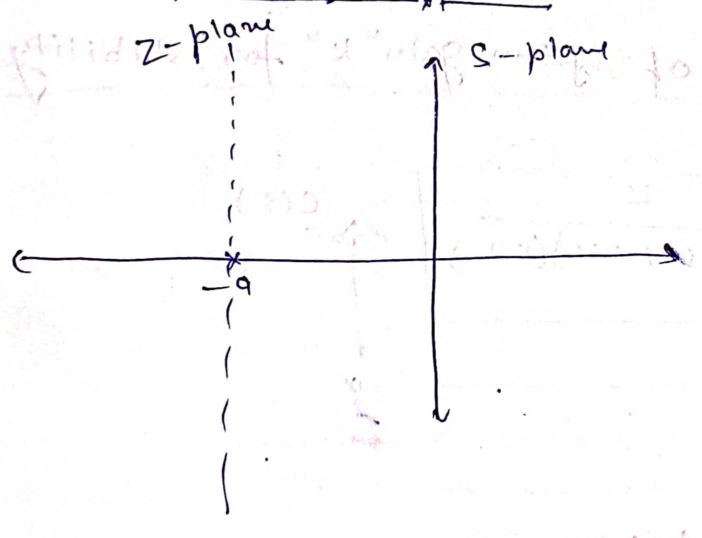
Ques 1.

$$P(s) = s^3 + 7s^2 + 25s + 39 = 0$$

Check whether the roots are lying more -vely w.r.t −1 ?

Sol:

Generalized form



$$s + a = z$$

$$\Rightarrow \boxed{s = z - a}$$

Now for the problems:-

As we have check for "−1".

∴ put $s = z - 1$

∴ $P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$

$\therefore P(z) = z^3 + 4z^2 + 14z + 20 = 0$

$\therefore$ Routh Array :-

z^3	1	14
z^2	4	20
z^1	9	0
z^0	20	0

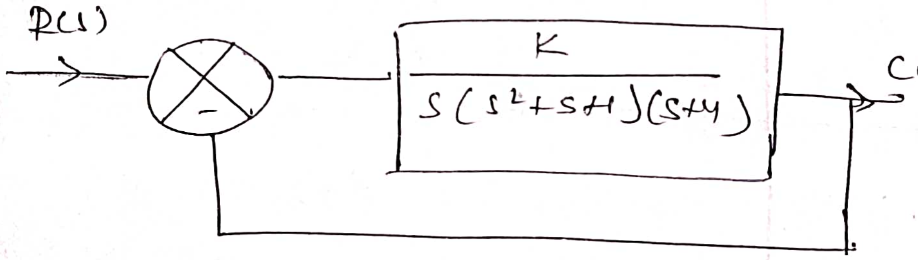
Now, as we see the I^{st} colomn is +ve,

$\Rightarrow$ The system is stable w.r.t Z.

$\therefore \Rightarrow$ The sys. is relatively more stable (for s-plane)

#→ Root locus :-

Range of values of system gain "k" for stability



$1 + G(s)H(s) = 0$

$(s^2 + s + s)(s+4) + k = 0$

$s^4 + 5s^3 + 5s^2 + 4s + k = 0$

Routh Array :-

s^4	1	5	K
s^3	5	4	0
s^2	$21/5$	K	0
s^1	$\frac{\frac{84}{5} - 5K}{21/5}$	0	0
s^0	K	0	0

For system is to be stable :-

i) $$\frac{\frac{84}{5} - 5K}{21/5} > 0$$

$\Rightarrow \frac{84}{5} - 5K > 0$

$\Rightarrow 5K < \frac{84}{5}$

$\Rightarrow K < \frac{84}{25} = 3.36$

$\Rightarrow \boxed{K < 3.36}$ —— (1)

(ii) and ii) $K > 0$

$\Rightarrow$ from (i) and (ii)

$$\boxed{0 < K < 3.36}$$

At $K = K_{marginal} = 3.36$

$s^1 = 0$.

$\therefore A(s) = \frac{21}{5} s^2 + K = 0$

$\Rightarrow \frac{21}{5} s^2 + 3.36 = 0$

$\Rightarrow s^2 = \frac{-3.36 \times 5}{21}$

$\Rightarrow s = \pm j0.9$

Now by ~~compa~~ comparing by $s = j\omega$

$$\boxed{\omega = 0.9 \ r/s}$$

Imp

#→ we can find the freq. of oscillation in case marginal stability only when ~~system~~ of Auxiliary eqn. is of order 2 only.

W.B

Ex:- 4

(8) :-

$$s^4 + 2s^3 + 3s^2 + 2s + K = 0$$

$\frac{4 - 2K}{2}$

s^4	1	3	K
s^3	2	2	0
s^2	2	K	0
s^1	2-K	0	0
s^0	K	0	0

from stability.

$2 - K > 0$

$\Rightarrow K < 2$ — (1)

and $K > 0$

$\Rightarrow 0 < K < 2$

$A(s) = 2s^2 + K = 0$

$2s^2 + 2 = 0$

$s = \pm j1$

$$\Rightarrow \boxed{\omega = 1 \ r/s}$$