UNIT - 1

Properties of Probability -

1 → the Probability of certain event (i.e. the event contains all the outcomes) is unity i.e. 1 $\boxed{P(A) = 1}$

2 → the probability of any event is always less than or equal to 1 and non-negative i.e. $\boxed{0 \leq P(A) \leq 1}$

3 → If A and B are two mutually exclusive events then
$$P(A+B) = P(A) + P(B)$$

4 → If A is any event, then the Probability of not happening of A i.e
$$P(\bar{A}) = 1 - P(A)$$

5 → If A and B are not mutually exclusive events then.
$$P(A+B) = P(A) + P(B) - P(AB)$$
$P(AB)$ = joint Probability of A & B

Conditional Properties of Probability :→

* the conditional ~~Propert~~ Probability of event B given that event A has already happend is given as. $$P(B/A) = \frac{P(AB)}{P(A)}$$
* similarly the conditional Probability of event A given that event B ~~as~~ already happened is given as. $$P(A/B) = \frac{P(AB)}{P(B)}$$

Bayes's Rule:→ Let $B_1, B_2, B_3 \ldots B_n$ be mutually exclusive events and event A ~~to~~ occures only when any one of $B_1, B_2, \ldots B_n$ occurs, then

$$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{\sum_{j=1}^{n} P(B_j)\,P(A/B_j)}$$

Q.1→ There are two identical boxes x & y. Box 'x' contains 4 white and 3 red balls and Box 'y' contains 3 white and 7 red balls. one ball is drawn at random from the box. if the ball is white, what is the probability that it is drawn from box 'x'.

Ans

B_1	B_2
$\boxed{x}$	$\boxed{Y}$
$7\begin{cases}4W\\3R\end{cases}$	$\begin{matrix}3W\\7R\end{matrix}$

$P(B_1) = P(B_2) = \frac{1}{2}$ Ans

$P(A/B_1) = \frac{4}{7}$ Ans

$P(A/B_2) = \frac{3}{10}$ Ans

Bayes's rule→

$$P(B_1/A) = \frac{P(B_1)\,P(A/B_1)}{P(B_1)\,P(A/B_1) + P(B_2)\cdot P(A/B_2)} = \frac{\frac{1}{2}\times\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7} + \frac{1}{2}\times\frac{3}{10}} = 0.65 \text{ Ans}$$

Q.20 Two factories produce identical clocks. the production for the first factory consists of 10,000 clocks of which 100 are defective. the second factory produces 20,000 clocks of which 300 are defective. what is the probability that a particular defective clock was produced in the first factory?

Ans factory Probability ⇒ Let 1st factory probability is B_1 ⇒ $P(B_1) = \frac{1}{2}$
and 2nd factory probability is B_2 ⇒ $P(B_2) = \frac{1}{2}$

Let select 1st factory then probability of defective clock.

$$P(A/B_1) = \frac{100}{10000} = \frac{1}{100}$$

similarly $P(A/B_2) = \frac{300}{20000} = \frac{3}{200}$

using Bayes's rule

$$P(B_1/A) = \frac{P(B_1) \cdot P(A/B_1)}{P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{100}}{\frac{1}{2} \times \frac{1}{100} + \frac{1}{2} \times \frac{3}{200}}$$

$P(B_1/A) = 0.4$ Ans

Random variables :- A function which takes on any value from the sample space and its range is some set of real numbers is called a random variable of the experiment. Random variables are two types.

(a) Discrete random variable → the discrete random variable has countable number of ~~disc~~ distinct values.

for example → $X = \{1, 4, 5, 7\}$ is discrete random variable.

(b) continuous Random variables ⇒ If random variable 'X' takes on any value is a whole observation interval, X is called a continuous random variable.

Commulative Distribution function (CDF) :- CDF of a random variable 'X' is the Probability that a random variable 'X' take place a value less than or equal to x. (x is any dummy variable)

$$\boxed{CDF \{ F_X(x) = P(X \leq x)}$$

Probability Density function :- (PDF) → the derivative of CDF with respect to some dummy variable 'x' is called a PDF.

$$f_X(x) = \frac{d\, F_X(x)}{dx}$$

Note ⇒ f = PDF & F = CDF

Properties of PDF:⇒

① $f_X(x) \geq 0$ for all value of x.

② the area under the PDF curve is equal to 1. i.e.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

③ $$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

④ $$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$$

Q.⇒ In an experiment, three coins are tossed simultaneously if the number of heads is the random variable, find the probability function for this random variable.

Ans⇒ total no. of Possibility $= 2^3 = 8$

$\{000, 001, 010, 011, 100, 101, 110, 111\}$

$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

3, 2, 2, 1, 2, 1, 1, 0

$P(X=1) = P(X=2) = P(X=3) = P(X=0) = ?$ (Random variable Probability)

$P(X=0) = \frac{1}{8}$, $P(X=1) = \frac{3}{8}$, $P(X=2) = \frac{3}{8}$, $P(X=3) = \frac{1}{8}$

X	0	1	2	3
$f(x)$	$1/8$	$\frac{3}{8}$	$3/8$	$1/8$

Q.2 Determine whether the following function is commulative distribution function (CDF) or not:-

$$F_X(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{1}{2}\left(\frac{x}{a}+1\right) & \text{for } -a \le x \le a \\ 1 & \text{for } x > a \end{cases}$$

Solⁿ: Convert this function in PDF By using Derivative of CDF.

$$f_X(x) = \begin{cases} 0 & x < a \\ \frac{1}{2a} & -a \le x \le a \\ 0 & x > a \end{cases}$$

Now using Properties of PDF, if the area of PDF is 1. then the given funⁿ must be CDF.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{-a} 0\,dx + \int_{-a}^{a} \frac{1}{2a}\,dx + \int_{a}^{\infty} 0\,dx = \int_{-a}^{a} \frac{1}{2a}\,dx = \left[\frac{x}{2a}\right]_{-a}^{a}$$

$= \int f_X(x)\,dx = \frac{a}{2a} + \frac{a}{2a} = \frac{2a}{2a} = 1$

Here area of PDF is unity so the given funⁿ is CDF.

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Q3 PDF is given by $f_X(x) = a e^{-b|x|}$. Here X is a random variable whose value lie in the range $x = -\infty$ to $x = +\infty$ Determine the following -

(1) Relationship b/w a & b.

(2) CDF

(3) Probability that outcomes lies b/w 1 & 2.

Solⁿ ⇒ As per properties $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ — ①

Put the value of f_X in eqⁿ ①

$$\int_{-\infty}^{\infty} a e^{-b|x|}\,dx = \int_{-\infty}^{0} a e^{-b(-x)}\,dx + \int_{0}^{\infty} a e^{-bx}\,dx$$

$$= \left[\frac{a e^{bx}}{b}\right]_{-\infty}^{0} + \left[\frac{a e^{-bx}}{-b}\right]_{0}^{\infty}$$

$$= \frac{a e^{0}}{b} - \frac{a e^{-\infty}}{b} + \frac{a e^{-\infty}}{-b} - \frac{a e^{-0}}{-b} \qquad \boxed{e^{-\infty} = 0}$$

$$= \frac{a}{b} + \frac{a}{b} = \frac{2a}{b} = \text{unity as per property}$$

So $\boxed{2a = b}$ is the relationship

(ii) Calculation of CDF $= F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

or $F_X(x) = \int_{-\infty}^{x} a e^{-b|x|}\,dx$ Break it because Here x is in MOD.

$$F_X(x) = \underbrace{\int_{-\infty}^{x} a e^{bx}\,dx}_{(x<0)} + \underbrace{\int_{0}^{x} a e^{-bx}\,dx}_{(x \geq 0)}$$

$$F_X(x) = \left[\frac{a e^{bx}}{b}\right]_{-\infty}^{x} + \left[\frac{a e^{-bx}}{-b}\right]_{0}^{x}$$

$$F_X(x) = \underbrace{\left[\frac{a e^{bx}}{b}\right]}_{x<0} + \underbrace{\left[\frac{a e^{-bx}}{-b} + \frac{a}{b}\right]}_{x \geq 0}$$

Put $2a = b$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{bx} & \text{For } x < 0 \\ \frac{1}{2} - \frac{1}{2} e^{-bx} & x \geq 0 \end{cases}$$

(iii) As Per Property $P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$

Now $P(1 \leq X \leq 2) = \int_{1}^{2} a e^{-b|x|}\,dx = \int_{1}^{2} a^{-1 \cdot x}\,dx$

$$= \left[\frac{a e^{-bx}}{b}\right]_1^2 = \frac{a}{b} e^{-2b} - \frac{a e^{-b}}{b}$$

using relation $2a = b$

$$= \frac{1}{2}\left[e^{-b} - e^{-2b}\right] \text{ Ans}$$

Statistical averages of Random Variable :→

(a) mean or Average value :→

i) the mean value of discrete random variable is

$$m_x = E(X) = \overline{X} = \sum_{i=1}^{n} x_i \, P(x_i)$$

ii) the mean value of continuous random variable is

$$\boxed{m_x = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx}$$ where $f_X(x)$ = Probability Density function.

(b) mean square value :→

$$\overline{X^2} = E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

(c) Variance :→

$$\text{variance of } X = E[(X - m_x)^2] = \int_{-\infty}^{\infty} (x - m_x)^2 f_X(x)\,dx$$

$\cdot X = E[(X-m_x)^2] = E[X^2 + m_x^2 - 2Xm_x]$

$= EX^2 + Em_x^2 - 2E(X)m_x$

But according to mean value $E(X) = m_x$

So $= Ex^2 + Em_x - 2m_x \cdot m_x$

Here m_x = constt.

So $= Ex^2 + m_x - 2m_x^2$

Variance of $\boxed{X = Ex^2 - m_x^2}$

i.e. variance of X = mean square value − mean square

d) ⇒ standard deviation :⇒

$\sigma_x = \sqrt{\text{Variance}} = \sqrt{E(X^2) - m_x^2}$

or $\sigma_x = \sqrt{(\text{mean square value}) - (\text{Square of mean Value})}$

Q.→ A random variable X has the uniform distribution given by.

$$f_X(x) = \begin{cases} \frac{1}{2\pi} & \text{for } 0 \le x \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

Determine $E[x]$, $E(x^2)$ and σ_x
(mean) (mean square value) (standard deviation)

Solution:

① $E[x] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

$$= \int_0^{2\pi} x \cdot \frac{1}{2\pi}\,dx$$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{2\pi}\left[\frac{4\pi^2}{2}\right]$$

Mean $E[x] = \pi$ Ans

② mean square value $E(x^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$

$$E(x^2) = \int_0^{2\pi} x^2 \frac{1}{2\pi}\,dx = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$E(x^2) = \frac{1}{2\pi} \times \frac{8\pi^3}{3} = \frac{4}{3}\pi^2 \text{ Ans}$$

② Now standard ~~variance~~ deviation.

$$\sigma_x^2 = E(x^2) - m_x^2$$

$$\sigma_x^2 = \frac{4\pi^2}{3} - \pi^2 = \frac{\pi^2}{3}$$

$$\sigma_x = \frac{\pi}{\sqrt{3}} \text{ Ans}$$

Q.→ Rayleigh Density function is given by

$$f_X(x) = \begin{cases} x e^{-\frac{x^2}{2}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

i) Find Distribution function $F_X(x)$

ii) $P(0.5 < x \leq 2)$

Solnﾠ→ Here give funn is PDF and CDF has to find.

So $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx = \int_{0}^{x} x \cdot e^{-\frac{x^2}{2}}\,dx$ ﾠﾠ Let $\frac{x^2}{2} = t$

$\frac{2x\,dx}{2} = dt$

$x\,dx = dt$

Now $F_X(x) = \int_{0}^{x^2/2} e^{-t}\,dt$

$$= \left[\frac{e^{-t}}{-1}\right]_0^{x^2/2} = -e^{-x^2/2} + e^{0} = 1 - e^{-\frac{x^2}{2}} \text{ Ans}$$

(iii) Let $P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f_X(x)\,dx$

so $P(0.5 < x \le 2) = \int_{0.5}^{2} x\, e^{-x^2/2}\, dx$

Let $\frac{x^2}{2} = t$

$\frac{2x}{2} dx = dt$

$x\,dx = dt$

$P(0.5 < x \le 2) = \int_{0.125}^{2} e^{-t} dt = \left[\frac{e^{-t}}{-1}\right]_{0.125}^{2}$

$= -e^{-2} + e^{-0.125}$

$= 0.747$ Ans

imp

Q ⇒ The PDF of a continuous random variable is of the form

$f_X(x) = \frac{1}{2} e^{-|x|}$ for $-\infty \le x \le \infty$ Determine mean and variance of random variables.

Solution ⇒ For ~~mean~~ mean $m_x = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

$m_x = \int_{-\infty}^{\infty} x \cdot \frac{1}{2} e^{-|x|}\, dx$

Let $\frac{x}{2} e^{-|x|} = c(x)$

Now let check the above function is even ~~ed~~ or odd function

if $x = -x$ then $-\frac{x}{2} e^{-|x|} = -c(x)$

and if $x = x$ then $\frac{x}{2} e^{-|x|} = c(x)$

Here it is clear that the above funⁿ is an odd funⁿ and the integration of odd funⁿ is always zero (0).

So $\int_{-\infty}^{\infty} c(x)\,dx = 0$

So $\boxed{m_x = 0}$ Ans

2) mean square value and variance

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_{-\infty}^{\infty} x^2 \frac{1}{2} e^{-|x|}\,dx$$

Again let $\frac{x^2}{2} \frac{1}{2} e^{-|x|}\,dx = c(x)$

Now the above function is even function

Here $c(x)$ is even funⁿ

So Property of even funⁿ $\int_{-\infty}^{\infty} f(x)\,dx = 2\int_{0}^{\infty} f(x)\,dx$

Now $E[X^2] = \int_{-\infty}^{\infty} x^2 \frac{1}{2} e^{-|x|} dx$

$= 2 \cdot \frac{1}{2} \int_0^{\infty} x^2 e^{-|x|} dx$

$= \int_0^{\infty} x^2 e^{-|x|} dx = \int_0^{\infty} x^2 e^{-x} dx$ — ①

We use the property ~~that~~ of Gamma function that

$\int_0^{\infty} x^{n-1} e^{-x} dx = \Gamma n$ (Gamma function of n) — ②

$= \Gamma n = (n-1)(\Gamma n-1)$

On comparing ~~comp~~ can be ~~a~~ written as.

$E[X^2] = \int_0^{\infty} x^{(3-1)} e^{-x} dx = \Gamma 3$

$\Gamma 3 = (3-1)\Gamma 2$

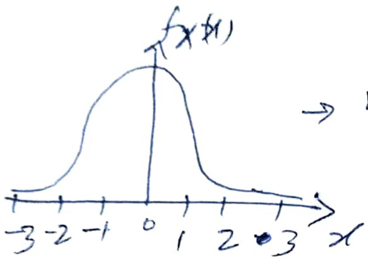
$= 2\Gamma 2 = 2(2-1)\Gamma 1 = 2 \cdot 1 \cdot 1 = 2$

So $\boxed{E[X^2] = 2}$

Now $\sigma_x^2 = E[X^2] - m_x^2 = 2 - 0 = 2$

$\boxed{\text{S.D} \quad \sigma_x = \sqrt{2}}$ Ans

Gaussian Random Variable: $\rightarrow$ A variable is called Gaussian Random variable when it's PDF is as given below.



$\rightarrow$ Normal distribution funn

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

when $-\infty < x < \infty$ and $\sigma > 0$

$-\infty < \mu < \infty$

the normal distribution is that distribution in which the mean and median is always zero

Notes Gaussian Random Process is define by the Gaussian random varriable $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

(a) mean Value of Gaussian Random Variable $\rightarrow$

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

Let $\left\{ \begin{array}{l} \dfrac{x-\mu}{\sqrt{2}\sigma} = t \quad \text{or} \quad x = \sqrt{2}\sigma t + \mu \\[2ex] \dfrac{dx}{\sqrt{2}\sigma} = dt \quad \text{or} \quad dx = \sqrt{2}\,\sigma\, dt \end{array} \right\}$

Now Putting these values. we get

$$E(X) = \int_{-\infty}^{\infty} (\sqrt{2}\,\sigma t + \mu) \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} \sqrt{2}\sigma\, dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \sqrt{2}\,\sigma t + \mu) e^{-t^2} dt$$

$$E[X] = \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} t\, e^{-t^2} dt}_{\text{odd fun}} + \frac{\mu}{\sqrt{\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-t^2} dt}_{\text{even fun}}$$

by using integration Property for even and odd fun.

$$E[X] = 0 + \frac{2\mu}{\sqrt{\pi}} \int_0^{\pi} e^{-t^2} dt$$

$$E[X] = \frac{2\mu}{\sqrt{\pi}} \int_0^{\pi} e^{t^2} dt$$

using substitution methode Let $t^2 = z \Rightarrow 2t\,dt = dz$ or $dt = \dfrac{dz}{2t}$

$$E[X] = \frac{2\mu}{\sqrt{\pi}} \int_0^{\infty} e^{-z} \frac{dz}{2\sqrt{z}}$$

$$E[X] = \frac{\mu}{\sqrt{\pi}} \int_0^\infty z^{-1/2} e^{-z} dz$$

now using Gamma function property of integration

$$\int_0^\pi x^{n-1} e^{-x} dx = \Gamma n$$

$$E[X] = \frac{\mu}{\sqrt{\pi}} \int_0^\infty x^{(1/2-1)} e^{-x} dx$$

$$= \frac{\mu}{\sqrt{\pi}} \Gamma 1/2$$ the value of $\Gamma 1/2 = \sqrt{\pi}$ calculated

$$= \frac{\mu}{\sqrt{\pi}} \cdot \sqrt{\pi}$$

$$\boxed{E[X] = \mu}$$

(b) Variance of Gaussian Random Variable: ⇒

We know that variance of $X = E[(X-m_x)^2] = \int_{-\infty}^{\infty} (x-m_x)^2 f_X(x) dx$

it means $E[(X-m_x)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma_X\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$

by substitution method

Let $\begin{cases} \dfrac{x-\mu}{\sqrt{2}\sigma} = t & \Rightarrow x = \sqrt{2}\sigma t + \mu \\ \dfrac{dx}{\sqrt{2}\sigma} = dt & \Rightarrow dx = \sqrt{2}\cdot\sigma\, dt \end{cases}$

By Putting these value we get

$$E[(X-m_x)^2] = \int_{-\infty}^{\infty} \sigma^2 \cdot 2t^2 \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-t^2}\, dt\, \sqrt{2}\,\sigma$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2}\, dt \quad \leftarrow \text{this is even fun}^n \text{ so}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot 2 \int_{0}^{\infty} t^2 e^{-t^2} dt$$

$$E[(X-m_x)^2] = \frac{4\sigma^2}{\sqrt{\pi}} \int_0^{\infty} t^2 e^{-t^2} dt$$

Again By substitution methode Put $t^2 = z$

$2t\,dt = dz$

$dt = \dfrac{dz}{2\sqrt{z}}$

$$E[(X-m_x)^2] = \frac{4\sigma^2}{\sqrt{\pi}} \int_0^{\infty} z e^{-z} \frac{dz}{2\sqrt{z}}$$

$$= \frac{4\sigma^2}{2\sqrt{\pi}} \int_0^{\infty} z^{1/2} e^{-z} dz$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} z^{1/2} e^{-z} dz$$

Now again gamma funⁿ.

using $\int_0^\infty x^{n-1} e^{-x} dx = \Gamma n$ Here $n-1 = \frac{1}{2}$

$= (n-1)\Gamma(n-1)$ $n = \frac{1}{2}+1 = \frac{3}{2}$

So

$$E[(X-m_x)^2] = \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\frac{3}{2}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} (\tfrac{3}{2}-1).\Gamma(\tfrac{3}{2}-1)$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \frac{1}{2}.\Gamma\frac{1}{2}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}.\frac{1}{2}.\sqrt{\pi}$$

$$\boxed{E(X-m_x)^2 = \sigma^2}$$

Now Standard deviation $= \sqrt{\text{Variance}} = \sqrt{\sigma^2} = \sigma$

(c) mean square value of Gaussian Random variable ⇒

$$\text{Variance} = \sqrt{E(X^2) - \text{mean}^2}$$

$$E[X^2] = \text{Variance}^2 + \text{mean}^2$$

$$\boxed{E[X^2] = \sigma^2 + \mu^2}$$

Random Process:⇒

* let there be a random experiment E having out come λ from the Sample space (S).

* If this out come λ is associated with time, then a function of λ and time ~~is~~ t is formed i.e. $X(\lambda, t)$

* this function $X(\lambda, t)$ is Known as random Process.

Random Process is just defined as a function that depends on the two variable one is λ which is out come from sample space experiment and it is depends ~~to~~ on the time value also. i.e $X(\lambda, t)$.

Statistical characteristics of random Process⇒

1) CDF $F_X(x,t) = P(X(t) \leq x)$

2) PDF $f_X(x,t) = \frac{d}{dt} F_X(x,t)$

3) mean $E\{X(t)\} = \int_{-\infty}^{\infty} x\, f_X(x,t)\, dx$

4) mean square $E\{X^2(t)\} = \int_{-\infty}^{\infty} x^2 f_X(x,t)\, dx$

5) Auto corelation funⁿ :-

$$R_X(t_1, t_2) = E[X(t_1) X(t_2)]$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy\, f_X(x, y; t_1, t_2)\, dx\, dy$$

or t is constt then

$$R_X(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x, y, f_X(x, y)\, dx\, dy$$ [Note: ⇒ 2 properties on next page.]

Types of Random Process :⇒

① Stationary Random Process ⇒ A random process is stationary if its statistics (mean, variance, etc) are not affected by any shift in time origin.

(2) Non-stationary random process :⇒ A process is non-stationary if its statistics are affected by any shift in time origin.

3) Wide sense stationary Random Process :⇒ the process may not be stationary in strict sense, still the mean and auto correlation function are independent of shift of time region.

Q → Consider the random process $x(t) = \cos(t+\phi)$ where ϕ is random variable with density function $f(\phi) = \frac{1}{\pi}$ where $-\frac{\pi}{2} \le \phi < \frac{\pi}{2}$. check whether or not the process is stationary.

solu→ for checking stationary we have to calculate the mean of given fun.

$$\boxed{m_{(t)} = \int_{-\infty}^{\infty} x\, f_X(x,t)\, dx} \quad \text{--- (1)}$$

Putting the value according to given question.

$$m_{(t)} = \int_{-\pi/2}^{\pi/2} \cos(t+\phi) \cdot \frac{1}{\pi}\, d\phi$$

$$= \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(t+\phi)\, d\phi = \frac{1}{\pi} \left[\sin(t+\phi)\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi} \left[\sin(\pi/2 + t) - \sin(t - \pi/2)\right]$$

$$= \frac{1}{\pi} \left[\sin(\pi/2 + t) + \sin(\pi/2 - t)\right]$$

$$m_{(t)} = \frac{1}{\pi} \left[\cos t + \cos t\right] = \boxed{\frac{2}{\pi} \cos t}$$

Here the mean value of given fun is dependent on t. so given fun is not stationary

Q. A stationary random process $X = \{X(t)\}$ has auto-correlation fusion $R(\tau) = 16 + 9e^{-|\tau|}$. Find the standard deviation of the process.

Solun → (a) $R(\tau) = E[X(t) \cdot X(t+\tau)]$ $\boxed{\text{if } \tau = 0}$

auto correlation properties :→

$R(0) = E[X(t) \cdot \cancel{X}(t)]$

$R(0) = E[X^2(t)]$ = mean square value

$R(0) = 16 + 9e^0 = 16 + 9 = 25 = E[X^2(t)]$

Now : (b) $\mu_x^2 = m_x^2 = \lim_{\tau \to \infty} R(\tau) = 16 + 9e^{-\infty} = 16$ ✓

$m_x^2 = 16 \Rightarrow m_x = 4$ = mean value.

Now (c) variance $= E[X^2(t)] - m_x^2$

Variance $= 25 - 16 = 9$

Now d) standard deviation $(\sigma) = \sqrt{\text{Variance}}$

$= \sqrt{9}$

$\sigma = 3$ Ans

Power spectral density:→ It is defined as fourier transform of auto correlation function. It is denoted by $S_X(f)$.

i.e. $$\boxed{S_X(f) = \int_{-\infty}^{\infty} R_X(t)\, e^{-j2\pi f t}\, dt}$$

Or using inverse fourier transform we find autocorrelation fun^n.

$$\boxed{R_X(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(f)\, e^{j2\pi f t}\, df}$$